Measurement of Polarization-Mode Dispersion

Polarization-mode dispersion is defined and characterized, using Poincaré sphere and Jones matrix concepts. Interferometric, wavelength scanning, and Jones matrix eigenanalysis measurement methods are described. Instrumentation, especially the HP 8509B lightwave polarization analyzer, is discussed.

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New generations of high-speed undersea telecommunication systems and cable TV distribution systems feature an important new player: the erbium-doped fiber amplifier, or EDFA. Moving quickly from laboratory to mainline application, the EDFA will lower the cost and increase the reliability of long-haul telecommunications and greatly increase head-end distribution power for CATV.

In contrast to older systems in which propagation loss was compensated by detecting the optical signal and retransmitting it at higher power (regeneration), the EDFA-based system is a continuous glass pathway with amplification provided at intervals by short lengths of pumped, erbium-doped fiber. The absence of pulse regeneration must be offset by improvements in the dispersive characteristics of the pathway.

Historically, polarization-mode dispersion, or PMD, is the third of a series of dispersive effects in optical fiber. The bandwidth of multimode fiber is limited because light separates into spatial modes of many different lengths. Single-mode fiber solves that problem but is limited by chromatic dispersion, in which the transmission medium allows adjacent wavelengths to travel at slightly different speeds. PMD, a more subtle effect, arises from slight physical asymmetry in the index of refraction, called birefringence. In fiber, it is caused by stresses induced by fiber manufacture, packaging, and deployment and is strongly influenced by environment.

When chromatic dispersion is sufficiently reduced, the pulse distortion and signal fading produced by PMD can be observed. In CATV systems, the combination of PMD in fiber and components, frequency chirp in the transmitter, and polarization dependent loss near the receiver produces composite second-order distortion. For high-speed, long-haul telecommunications, and high-channel-capacity CATV systems to realize their potential, PMD must be understood and controlled.

Polarization-mode dispersion is a fundamental property of single-mode optical fiber and components in which signal energy at a given wavelength is resolved into two orthogonal polarization modes of slightly different propagation velocity. The resulting difference in propagation time between polarization modes is called the differential group delay, commonly symbolized as $\Delta \tau_g$, or simply $\Delta \tau$. In most optical components, the polarization modes correspond to physical axes of the component and the differential group delay (and therefore the PMD) is nearly independent of wavelength. In practical lengths of optical fiber, differential group delay varies randomly with wavelength and the specification of PMD must be statistically based. Long-fiber PMD is commonly expressed as either the average value or the rms value of differential group delay over a wide wavelength range. For fibers that exhibit a large degree of coupling of energy between polarization modes, PMD scales with the square root of fiber length and is often specified in picoseconds per root kilometer.

How much PMD is too much? For modest impact, the instantaneous differential group delay of a telecommunication system must be kept below one tenth of a bit period, or 20 ps for a 5-Gbit/s NRZ pulse stream.

Characterizing PMD

PMD in many real systems varies over time and is best characterized by a statistical picture to account for its changing details. For the moment, however, let’s consider how to characterize the PMD of a stable device or system that exhibits no time variation. Differential group delay, the most direct measure of the signal-distorting effects of PMD, does not tell the whole story. The PMD of a system is completely characterized by specifying any of the following three quantities as a function of wavelength or optical frequency:

- A pair of principal states of polarization and a differential group delay
- A three-dimensional polarization dispersion vector
- A Jones matrix (see page 28).

If a polarized, tunable optical wave is transmitted through a device, the polarization at the device output will in general trace out an irregular path on the Poincaré sphere (see page 29) as the optical radian frequency $\omega$ is tuned, as shown in Fig. 1. Over a small range of frequency, any section of the irregular path can be approximated as an arc of a circle on the surface of the sphere. The center of such a circle, projected to the surface of the sphere, locates a principal state of polarization. A second, orthogonal principal state of polarization is located diametrically opposite on the sphere. The principal states of polarization are significant because they summarize how any output state of polarization evolves with frequency. As a function of frequency, all output states
Jones Calculus

Between 1941 and 1948, R. Clark Jones published a series of papers describing a new polarization calculus based upon optical fields rather than intensities. This approach, although more removed from direct observation than previous methods, allowed calculation of interference effects and in some cases provided a simpler description of optical physics. A completely polarized optical field can be represented by a two-element complex vector, each element specifying the magnitude and phase of the $x$ and $y$ components of the field at a particular point in space. The effect of transmission through an optical device is modeled by multiplying the input field vector by a complex two-by-two device matrix to obtain an output field vector.

The matrix representation of an unknown device can be found by measuring three output Jones vectors in response to three known stimulus polarizations. Calculation of the matrix is simplest when the stimuli are linear polarizations oriented at 0, 45, and 90 degrees (Fig. 1), but any three distinct stimuli may be used. The matrix calculated in this manner is related to the true Jones matrix by a multiplicative complex constant $c$. The magnitude of this constant can be calculated from intensities measured with the device removed from the optical path, but the phase is relatively difficult to calculate, requiring a stable interferometric measurement. Fortunately, measurements of many characteristics such as PMD do not require determination of this constant.

![Fig. 1](image)

**Fig. 1.** Relationship between the polarization dispersion vector $\mathbf{\Omega}$ and the output state of polarization $\mathbf{s}$ on the Poincaré sphere. The heavy line shows the path of the output polarization as the optical frequency changes. The output path is approximated over a small range of frequency as a circular arc generated by rotation about the polarization dispersion vector. The vector points in the direction of one principal state of polarization, and a second principal state of polarization is located diametrically opposite on the sphere.

The length of $\mathbf{\Omega}$ is the differential group delay. When the output state of polarization is expressed as a three-dimensional vector $\mathbf{s}$ composed of normalized Stokes parameters located the state of polarization on the sphere, the rotation about the principal states of polarization can be written as a cross product relation: $ds/d\omega = \mathbf{\Omega} \times \mathbf{s}$. In the most general case $\mathbf{\Omega}$ is a function of the optical frequency $\omega$.

In some optical components, such as isolators and waveguide modulators, PMD originates in crystals or waveguides through which light propagates with different group delays for different polarizations. As the optical frequency transmitted through these components is varied, $\mathbf{\Omega}$ remains fixed in orientation, while $|\mathbf{\Omega}|$ might vary slightly as a result of chromatic dispersion in each polarization mode. Devices such as these, in which $\mathbf{\Omega}$ is essentially independent of $\omega$, are especially simple to describe and to measure. In particular, the differential group delay is constant over time and can be accurately characterized as a weak function of frequency. Such devices can be useful as standards because their characteristics can be expected to remain stable from one time and location to the next.

Modern single-mode fibers achieve a very low level of local birefringence in addition to low propagation loss, but the effects of small local birefringences along the fiber can accumulate to cause significant PMD through a fiber many kilometers long. Birefringence is a property of a dielectric describing the difference in the indexes of refraction for different polarizations. Local birefringence in a fiber can be caused by deviations from perfect circular symmetry of the fiber core, or by asymmetrical stress in the core region owing to the manufacturing process, bends in the fiber, or temperature gradients. Stresses can change with temperature and with time as the glass and the surrounding cable relax, leading to a birefringence along the fiber length that evolves.

rotate about a diameter connecting the two principal states of polarization. The rate of rotation is determined by the differential group delay.

The polarization dispersion vector $\mathbf{\Omega}$ is probably the most intuitively meaningful representation of PMD because it is defined in the same real, three-dimensional space as the Poincaré sphere. This vector originates at the center of the Poincaré sphere and points toward the principal state of polarization about which the output states of polarization rotate in a counterclockwise sense with increasing $\omega$. $|\mathbf{\Omega}|$, the
over time. This behavior leads to measurable PMD that is a function of both frequency and time, so that a statistical picture becomes the most appropriate view of PMD for the system designer.

A detailed statistical model of PMD has been developed from the basis of accumulated local birefringences, and has been experimentally confirmed.\(^\text{5,4}\)

Each of the \(x\), \(y\), and \(z\) components of the polarization dispersion vector for a long fiber follows a normal distribution with zero mean. As a result, the orientation of \(\Omega\) is uniformly distributed, and the distribution of the differential group delay \(\Delta t = [\Omega]\) is proportional to \(\Delta t^2 \exp(-\Delta t^2 / 2 \sigma^2)\). This is often called a Maxwell distribution because it is the same as the Maxwell distribution of molecular speed for a gas in thermal equilibrium. The distribution has an expected value \(\langle \Delta t \rangle\) of \(\alpha \sqrt{8/\pi}\).

The statistical theory predicts, and experiments confirm, that the differential group delay distribution measured at a particular frequency over a long period of time is identical to the distribution measured at one time over a large range of frequency. This fact allows statistics representing slow time variations to be measured very quickly by gathering data over a wide frequency range. As another result of the statistical model, when several long fiber sections are concatenated, the expected value of differential group delay for the concatenation is given by the root sum of squares of the expected values for the sections, that is,

\[
\langle \Delta t \rangle_{\text{total}} = \sqrt{\langle \Delta t \rangle_1^2 + \langle \Delta t \rangle_2^2 + \ldots}.
\]

As a consequence, \(\langle \Delta t \rangle_{\text{total}}\) grows proportionally to the square root of the fiber length, and the PMD of a long fiber is specified in units of ps/\(\sqrt{\text{km}}\) with the understanding that the orientation of \(\Omega\) is uniformly distributed. In contrast, the PMD of a short section of fiber or of a fiber manufactured with a consistent birefringence over its length is specified in units of ps/km because it grows proportionally to the fiber length, and the orientation of \(\Omega\) is understood to be fixed relative to the physical orientation of the fiber. PMD in components is typically not statistical in origin, and is simply specified in ps.

Measuring PMD

Two polarization modes are transmitted through a device exhibiting significant PMD, each according to its own phase delay and group delay. Owing to the unequal group delays, propagation through such a device will change the mutual temporal coherence between the two polarization modes.
Likewise, the two unequal phase delays lead to a frequency dependent output state of polarization in response to a fixed input state of polarization. These physical characteristics make possible a variety of PMD measurement methods. An interferometric method\textsuperscript{5,6} measures the effect of PMD on mutual coherence, and a wavelength scanning method\textsuperscript{7} measures the effect of PMD, through variations of the output state of polarization, on transmission through a fixed analyzer. A method developed by Hewlett-Packard calculates the differential group delay and principal states of polarization as a function of frequency by analyzing Jones matrices measured at a sequence of optical frequencies.\textsuperscript{8,9,10} Most of the techniques currently used to measure PMD are similar in principle to one of these three methods.

A block diagram of the low-coherence interferometric method is shown in Fig. 2a. Collimated light from a broadband light-emitting diode is polarized and split into two mutually coherent beams. One mirror can be scanned in position, creating a differential delay between the two orthogonal polarizations, which are recombined and directed through the device under test (DUT). When photocurrent is measured as a function of the differential interferometer delay, coherent fringes can be observed only when this differential delay is compensated by the differential group delay of the DUT. Fig. 2b shows the envelope of the coherent fringes measured as a function of delay.

The wavelength scanning method, also called the fixed analyzer method, is shown schematically in Fig. 3a and can be assembled using equipment found in many optics laboratories without specialized equipment for PMD measurement. Polarized broadband light is directed through the DUT. PMD in the DUT causes the output state of polarization to trace out an irregular path on the Poincaré sphere when measured as a function of optical frequency, as was shown in Fig. 1.

By measuring the optical power transmitted through an output analyzer as a function of optical frequency (Fig. 3b), we effectively measure one dimension of the three-dimensional path on the sphere, leading to ripples in the spectral density measured by the optical spectrum analyzer. The average differential group delay over the measured frequency span is proportional to the number of spectral density extrema within the span.

Jones matrix eigenanalysis\textsuperscript{8} is based upon Jones matrices measured at a sequence of optical frequencies using the HP 8509B lightwave polarization analyzer and the HP 8167A and 8168A tunable laser sources,\textsuperscript{11} as shown schematically in Fig. 4a. At each frequency a Jones matrix $T_k$ is measured by stimulating the DUT with three accurately known states of polarization and measuring the response state of polarization at the DUT output.\textsuperscript{12} The matrix product $T_{k+1}T_k^{-1}$ reveals the change in the polarization transformation caused by the change in frequency, which in this case is a rotation about $\Omega$. The eigenvectors of $T_{k+1}T_k^{-1}$ yield the orientation of $\Omega$ and
the eigenvalues $\rho_1$ and $\rho_2$ of $\mathbf{T}_{k+1}\mathbf{T}_{k}^{-1}$ yield the differential group delay through the relation:

$$\Delta \tau = \left| \mathbf{\Omega} \right| = \frac{\text{Arg}[\rho_1/\rho_2]}{\Delta \nu},$$

where $\text{Arg}$ is the argument function (the argument of a complex number is its polar angle, that is, $\text{Arg} e^{i\beta} = \beta$). Stepping pairwise through the sequence of matrices $\mathbf{T}_k$, we obtain both principal states of polarization and the differential group delay as a function of optical frequency (Fig. 4b).

The ability to measure the frequency dependence of PMD can be helpful in diagnosing its physical origins, but in evaluating a highly mode-coupled device such as a long fiber only the average differential group delay is significant. This raises the question of how much data must be collected before a reliable estimate of the average differential group delay $\langle \Delta \tau \rangle$ is obtained. All three of these techniques gather data over a range of frequencies, and larger frequency or wavelength spans generally result in more reliable estimates of $\langle \Delta \tau \rangle$.

The interferometric and Jones matrix methods both can give an indication of when sufficient data has been collected by comparing measured data to curves theoretically predicted by the statistical model. Interferometry measures data we expect to be normally distributed, so the measured data is compared to a Gaussian curve to assess the estimate of $\langle \Delta \tau \rangle$. Fig. 5a shows a good fit. Eigenanalysis measures the magnitude $|\mathbf{\Omega}|$, which we expect to follow a Maxwell distribution. The curve of differential group delay versus wavelength is first converted into a histogram showing the number of measurements versus differential group delay, and then the histogram is compared to a Maxwell curve. Fig. 5b shows a good fit.

When the frequency span is not sufficient to produce a sufficiently good fit to the expected curve, new data can be collected by waiting for the physical properties of the fiber to change, assuming that the same statistical model remains valid. In the laboratory the characteristics of a spooled fiber are measured again after the fiber temperature is changed by a few degrees, while a deployed fiber must be measured again after several hours have elapsed. Multiple measurements over the same frequency span allow collection of a set of data that accurately predicts $\langle \Delta \tau \rangle$, as reflected by a good fit to the expected curve.

**Instrumentation**

The HP 8509B lightwave polarization analyzer, discussed on page 32, provides two automated methods for measurement of PMD: the Jones matrix eigenanalysis and three-Stokes-parameter wavelength scanning methods (Fig. 6). Both make use of the HP 8167/68A tunable laser sources.\(^{11}\) The
The HP 8509A/B Lightwave Polarization Analyzer

With the advent of the lightwave polarimeter, engineers in the fields of high-speed telecommunications, cable TV distribution, optical sensing, optical recording, and materials science can characterize polarization phenomena with the ease and graphical simplicity of the common oscilloscope. Supplemented by an optical source, a polarization state generator, and comprehensive measurement software, the polarimeter becomes a polarization analyzer, producing comprehensive measurements of both optical signals and two-port optical devices.

The HP 8509B lightwave polarization analyzer consists of an optical unit and a 66-MHz HP Vectra PC. The main display window, shown in Fig. 1, conveniently displays the polarization parameters of an optical signal and provides access to commonly used controls. The Measurements menu provides access to a variety of integrated measurement solutions addressing polarization-mode dispersion (PMD), polarization dependent loss, the Jones matrix, and optimization of optical launch into polarization maintaining fiber. The Display menu allows customization of the display window and the System menu enables the user to reconfigure system operating parameters, optimize performance at a particular wavelength, and automatically check the functional integrity of the instrument.

The heart of the HP 8509B is a high-speed polarimeter (see Fig. 4a in the accompanying article). A passive optical assembly (see cover) divides the optical signal into four beams and passes each beam through polarization filters to photodiode detectors. Autoranging amplifiers and 16-bit ADCs complete the circuitry. A series of calibration coefficients are determined at manufacture and stored in UV-PROM. The instrument interpolates among these coefficients to provide operation from 1200 to 1600 nm. Parallel filtering and detection combined with high-speed conversion and computation result in a measurement rate of 3000 polarization states per second.

A second optical assembly inserts three polarizing filters in the optical source path to allow measurement of the Jones matrix. The Jones matrix eigenanalysis PMD measurement method is based upon Jones matrices (see page 28) measured at a series of wavelengths. Polarization dependent loss is also derived from the Jones matrix. In addition, the user can use external polarizers to define a physical reference frame, analytically removing the birefringence and polarization dependent loss of components between the polarizer and the polarimeter receiver. Once defined, the reference frame allows the measurement of absolute polarization state at a point far from the instrument itself.

The wavelength scanning method determines three PMD values from changes in output polarization as observed along the three axes of the Poincaré sphere, then averages these results to provide a single value of PMD that is much less dependent on launch condition than conventional implementations of wavelength scanning. The measured normalized Stokes parameters are independent of signal power and are therefore immune to optical signal level changes, allowing a better measurement to be derived from a single wavelength sweep. Because the wavelength scanning method does not require the internal three-state polarizer, the method is also available on the HP 8509A.

The HP 71450A optical spectrum analyzer with Option 002 (internal white light source) is a powerful foundation for traditional wavelength scanning PMD measurements. The interferometric method of PMD measurement is available for certain applications via the HP 8504A/B precision reflectometer.

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References

Fig. 6. HP 8509B lightwave polarization analyzer display of polarization-mode dispersion (PMD) measurement results. The Jones matrix eigenanalysis method (foreground) measures differential group delay (DGD) as a function of wavelength. The three-Stokes-parameter wavelength scanning method (background) is enhanced by analyzing three responses produced by a single wavelength sweep.