

A Decision Support System for Integrated Circuit Package Selection

The package provides signal and power distribution, heat dissipation, and environmental protection for an integrated circuit (IC). The process of selecting a package is complicated by the large number of packaging alternatives with overlapping capabilities. To handle these difficulties, a decision support system was developed. The Package Selection System (PASS) combines expert system tools and multiple-attribute decision making techniques. The expert system provides a list of technically feasible alternatives. The multiple-attribute decision making modules are used to rank the alternatives based on nontechnical criteria.

by Craig J. Tanner

Two current trends in the electronic industry greatly increase the need for tools that assist in the selection of IC packages. The first trend is the emergence of a new field of electronics known as “manufacturingless manufacturers” (MLMs). The second trend involves the introduction of competing and overlapping technologies from subcontract packaging services. This trend complicates the package selection process because it creates many technically feasible packages for the decision maker to choose from.

MLMs are only concerned with the design phase of IC manufacturing, and in some cases final package test. All of the support activities associated with ICs, such as wafer fabrication, test, and packaging of the IC are subcontracted outside of the company to foundry services. The finished product is then sold by the MLM to the end user. MLMs have several advantages over full-service semiconductor companies. They can concentrate on doing one thing (design) well and they have no overhead or R&D costs associated with maintaining and developing IC manufacturing processes. Because the engineers who work for MLMs are typically focused on the design phase and are not experts in packaging, an IC package selection system can be a valuable tool.

The introduction of competing technologies from packaging subcontractors has made the selection process more difficult. In the past, there was usually one dominating technology that filled a particular niche. As United States and Asian subcontractors have become more involved in the research and development of new technologies, sometimes several packages that have similar technical attributes have been made available to users almost simultaneously. For example, the packages shown in Table I were released within 18 months of each other and all were aimed at users who need thermally enhanced surface mount packages.

Multiple-attribute decision making (MADM) techniques are incorporated into the decision support system to provide the decision maker with a method for selecting an IC package from among technically feasible alternatives. The MADM modules allow the decision maker to rank the feasible alternatives using nontechnical criteria. This technique can effectively address the emerging trend of simultaneous introduction of competing technologies from different suppliers.

Table I
Competing Thermally Enhanced Surface Mount Technologies

Technology	Company
Metal Quad Flat Pack	OIT
Micro Cool	Motorola
Enhanced Dissipative Quad Flat Pack	ASAT
Power Quad I & II	Anam

System Overview

The Package Selection System (PASS) contains all of the subsystems that are typical components of a decision support system:¹

- Database management system (DBMS)
- Model-base management system (MBMS)
- Dialog generation and management system (DGMS).

The DBMS is composed of a database and a management system. A database is a file or set of files containing information needed, generated, or manipulated by a computer program. The management system provides the method for creating, accessing, and maintaining the database. The PASS database is an *individual records model*. This model consists of a set of records in which each record contains a set of fields. Each record represents a type of package. Attributes of that package, such as price, are contained in the fields. Because the database is an ASCII file, a separate management system is not required. The database can be updated and maintained through the use of any text editor or word processor that is capable of reading and writing ASCII files.

The MBMS consists of three models. The first model is a knowledge-based expert system. The expert system is used to determine which IC packages are technically feasible. The second and third models use the multiple-attribute decision making techniques known as PROMETHEE and AHP. These models allow the decision maker to rank the technically feasible alternatives based on pricing and other nontechnical attributes.

The DGMS is classified as a *graphical user interface*. A graphical user interface (GUI) is a means of visually communicating between the user and the application. The attributes of visual communication include the graphical techniques used to communicate a concept, a task, a message, the contents of a screen, or any other interface component. The overall look and feel of a GUI is established through the use of screens, windows, controls, panels, icons, menus, animation, and sometimes sound (while sound is not visual, it can be a visual enhancer).

A sample screen from PASS is shown in Fig. 1. This screen is used for entering custom alternatives. Output from PASS sessions is presented in two ways: graphically and as an ASCII text file. The DGMS also contains an online help facility.

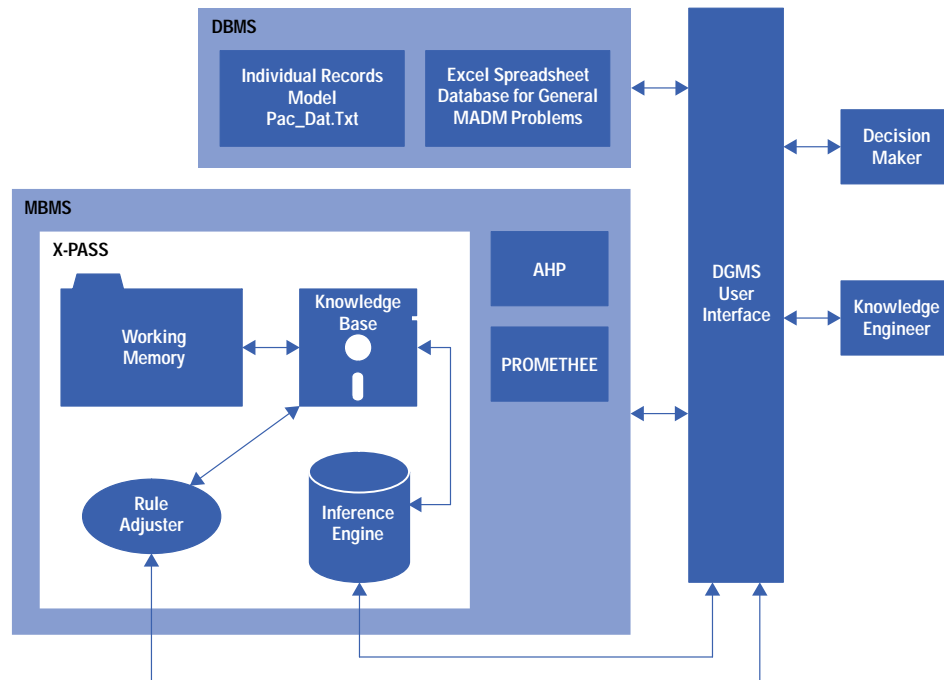
Fig. 1. Package Selection System (PASS) screen used for entering custom alternatives.

Attribute	Value
Price (\$)	11.25
Design (Wks)	6
Lead Time (Wks)	6
Support (hrs)	5
Sourcing (#)	7
Quality	5
Rework	10
Prototyping	10

Enter

The three subsystems described above must interface with each other and the decision maker to form a complete decision support system. A block diagram of the PASS components is shown in Fig. 2. The MBMS contains analytical tools and heuristic tools. The AHP and PROMETHEE modules are mathematical models while X-PASS employs expert heuristics in the form of a knowledge base and an inference engine. Decision support systems that use both of these problem-solving methods are frequently called *hybrid decision support systems*.²

Fig. 2. PASS block diagram.



Multiple-Attribute Decision Making

Multiple-attribute decision making (MADM) is the study of techniques that can be used by a decision maker to select a good alternative from a finite number of alternatives when faced with conflicting objectives. MADM techniques are necessary in PASS because X-PASS typically generates multiple alternatives and a method is needed to evaluate these alternatives. X-PASS only makes recommendations based on the technical aspects of electronic packaging. Technical attributes tend to overlap packaging technologies.

Nontechnical attributes such as price and delivery have varying degrees of importance based on the application and the decision maker's objectives for a particular integrated circuit. MADM techniques offer powerful ways of dealing with the decision maker's preferences and for ranking alternatives. For this reason multiple-attribute decision making has been included in PASS.

PASS contains two modules for performing multiple-attribute decision making. The first module uses PROMETHEE (Preference Ranking Organization Methods for Enrichment Evaluation). The second module uses AHP (Analytic Hierarchy Process).

Including both PROMETHEE and AHP in the decision support system allows those users who subscribe to either the French or the American schools of thought to take advantage of that preference. Offering both techniques is also an advantage for those users who have no preference or are not familiar with MADM techniques. These users benefit by examining both methods and selecting the technique with which they are most comfortable. The following sections contain a brief discussion of the PROMETHEE and AHP methods.

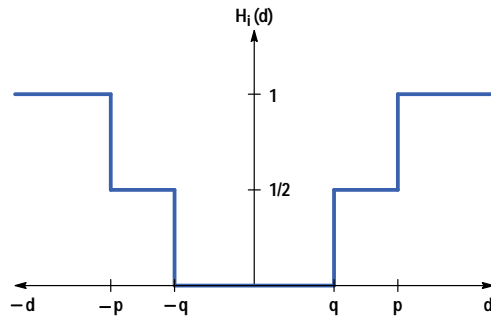
PROMETHEE

The PROMETHEE technique was presented by Brans and Vincke in 1985.³ PROMETHEE is an outranking method and can result in the partial preordering of alternatives (PROMETHEE I) or the complete preordering of alternatives (PROMETHEE II). The PROMETHEE methods consist of three steps: construction of generalized criteria or preference functions, calculation of the multicriteria preference index, and determination and evaluation of an outranking relation to give an answer to the multiple-attribute problem of interest.

During the construction of generalized criteria, the decision maker must assign a preference function, $P_h(a,b)$, to each criterion f_h , where $h = 1,2,\dots,k$. The preference function gives the degree of preference of the decision maker for selecting action a rather than action b. Four meanings are given to the preference function:

- $P_h(a,b) = 0$ No preference for a over b, $f_h(a)$ and $f_h(b)$ indifferent.
- $P_h(a,b) \sim 0$ Weak preference for a over b, $f_h(a) > f_h(b)$.
- $P_h(a,b) \sim 1$ Strong preference for a over b, $f_h(a) \gg f_h(b)$.
- $P_h(a,b) = 1$ Strict preference for a over b, $f_h(a) \gg \gg f_h(b)$.

Fig. 3. Graph of the generalized criterion $H_i(d)$. The parameters p , q , and d are defined in the article.



For example, if in comparing a \$5.00 package a to a \$7.00 package b, the decision maker feels that the \$2.00 difference between $f_h(a) = \$5.00$ and $f_h(b) = \$7.00$ is insignificant, then there is no preference for a over b and $P_h(a,b) = 0$.

The difference between the two evaluations, d , is equal to $f_h(a) - f_h(b)$. $H_h(d)$ is then defined as:

$$H_h(d) = P_h(a,b), \quad d \geq 0$$

$$= P_h(b,a), \quad d \leq 0.$$

The function $H_h(d)$ combined with the criterion f_h , that is, $\mathbf{H}_h(d) = \{H_h(d), f_h\}$, is called the *generalized criterion* associated with f_h . Brans, Vinke, and Mareschal⁴ have developed six possible types of generalized criteria. While other generalized criteria can be defined, these six should meet most decision makers' needs. They require that the decision maker define only a few parameters.

One of these generalized criteria is defined as follows (see Fig. 3):

$$\mathbf{H}_i(d) = 0, \quad |d| \leq q$$

$$= 1/2, \quad q < |d| \leq p$$

$$= 1, \quad \text{otherwise,}$$

where q is the indifference threshold, which represents the largest value of d below which the decision maker considers there is indifference, and p is a strict preference threshold, which represents the lowest value of d above which the decision maker considers there is strict preference.

The second step of the PROMETHEE method is to calculate the multicriteria preference index for each alternative over all criteria. The preference index is defined as:⁴

$$\pi(a, b) = \sum_{h=1}^k W_h P_h(a, b),$$

where

$$\sum_{h=1}^k W_h = 1$$

and

$$0 \leq \pi(a,b) \leq 1.$$

This index is the mean of the values of the k preference functions. W_h is a weight associated with each criterion. A weak preference of a over b is denoted by the value of $\pi(a,b)$ being close to zero. A strong preference of a over b is denoted by the value of $\pi(a,b)$ being close to one.

The third and final step of the PROMETHEE method, determination and evaluation of an outranking relation, requires that positive and negative outranking flows be determined from the multicriteria indexes. The positive and negative flows are defined as:³

$$\Phi^+(a) = \sum_{b \in A} \pi(a, b)$$

and

$$\Phi^-(a) = \sum_{b \in A} \pi(b, a),$$

where A is the set of possible actions.

The following rules can then be used to generate a partial preordering of the alternatives. P means “is preferred to,” I means “is indifferent to,” and iff means “if and only if.”

- aP^+b iff $\Phi^+(a) > \Phi^+(b)$
- aI^+b iff $\Phi^+(a) = \Phi^+(b)$
- aP^-b iff $\Phi^-(a) < \Phi^-(b)$
- aI^-b iff $\Phi^-(a) = \Phi^-(b)$.

The *outranking relationship* is then constructed from these rules:

- a outranks b if aP^+b and aP^-b or aP^+b and aI^-b or aI^+b and aP^-b .
- a is indifferent to b if aI^+b and aI^-b .
- a and b are incomparable otherwise.

Analytic Hierarchy Process

AHP was developed by Thomas L. Saaty in the 1970s. It is based on a set of axioms developed by Saaty⁵ and published in a paper by Harker and Vargas in 1987.⁶ A good mathematical analysis of these axioms can be found in reference 7. The four axioms were paraphrased by Harker as follows.

1. Given any two alternatives i and j out of the set of alternatives A, the decision maker is able to provide a pairwise comparison a_{ij} of these alternatives under any criterion c from the set of criteria C on a ratio scale that is reciprocal, that is,

$$a_{ji} = \frac{1}{a_{ij}} \quad \text{for all } i, j \in A.$$

2. When comparing any two alternatives $i, j \in A$, the decision maker never judges one to be infinitely better than another under any criterion c, that is, $a_{ij} \neq \infty$ for all $i, j \in A$.
3. One can formulate the decision problem as a hierarchy.
4. All criteria and alternatives that impact the given decision problem are represented in the hierarchy, that is, all expectations must be represented (or excluded) in terms of criteria and alternatives in the structure and be assigned priorities that are compatible with expectations.

Using these axioms, decision applications of AHP can be carried out in two phases: hierarchy design and evaluation.

The first phase, hierarchy design, involves the estimation of weights for each criterion that is used to rank alternatives. Most decision makers are faced with two types of criteria: quantitative and qualitative. For criteria based on quantitative data such as cost or size, the weights can be estimated by normalizing or inversely normalizing the comparison factors for each column of alternatives such that the weights sum to 1:

$$w_{ij} = \frac{a_{ij}}{\sum_{k=1}^n a_{kj}} \quad \text{for all } i, j = 1, 2, \dots, n.$$

For criteria based on qualitative data, a relative weight matrix can be constructed using Saaty’s scale of measurement⁶ (see Table II).

The positive reciprocal matrix constructed using a verbal scale technique may contain errors in judgment.⁴ Column normalization of this type of data would produce different results depending on which column was chosen.

Saaty’s eigenvector method⁸ is one technique for generating weights that effectively deals with these errors. The eigenvector method results in final weights that are an average of all possible ways of comparing the alternatives. The weights from the eigenvector method are calculated by raising the matrix of alternatives $\mathbf{A} = \{a_{ij}\}$ to increasing powers of k and then normalizing the resulting system:⁴

$$w = \lim_{k \rightarrow \infty} \frac{\mathbf{A}^k \mathbf{e}}{\mathbf{e}^T \mathbf{A}^k \mathbf{e}},$$

where \mathbf{e} is a column vector consisting of all 1s and \mathbf{e}^T is the transpose of \mathbf{e} . When w converges, the process is complete and a consistency index can be calculated that is an indication of the magnitude of the errors in the matrix.

After construction of the hierarchies, the second phase of AHP is evaluating the hierarchies to make a decision. Evaluation begins with constructing a final hierarchy of the pairwise comparisons of the criteria. Because the criteria are not usually equally important or quantifiable, Saaty’s scale of measurement and eigenvector approach are well-suited to developing the weights for ranking the importance of the criteria. The order of preference can then be determined by summing the relative priorities by weighting them with the overall priority of the given criterion.

Table II
Saaty's Scale of Measurement

Value	Definition
1	Equally important or preferred
3	Slightly more important or preferred
5	Strongly more important or preferred
7	Very strongly more important or preferred
9	Extremely more important or preferred
2,4,6,8	Intermediate values to reflect compromise
Reciprocals	Used to reflect dominance of the second alternative over the first

Using PASS

The Package Selection System is a Microsoft Windows[®]-based application. It was developed using Microsoft's Visual BASIC, version 2.0. Visual BASIC is a programming methodology that allows the developer to create programs that can take advantage of the Windows graphical user interface (GUI). Visual BASIC applications have the overall "look and feel" of professional Windows applications including pull-down menus, buttons, check boxes, scroll bars, text boxes, and icons. Visual BASIC programs can take advantage of other Windows features including multiple-document interface (MDI), dynamic data exchange (DDE), object linking and embedding (OLE), and dynamic link libraries (DLL).

A typical PASS session is run in the following manner:

1. Select technically feasible alternatives using the expert system, X-PASS.
2. Start the multiple-attribute decision making module by clicking on MADM.
3. Input the number of criteria and alternatives.
4. Select the MADM tool (AHP or PROMETHEE).
5. Input the alternatives. Usually alternatives are selected from those suggested by X-PASS but the user is free to select other alternatives or define a custom alternative.
6. Input the criteria. Several criteria are suggested by PASS for the purpose of ranking alternatives, but the user is free to define custom criteria.
7. Input the information required to describe the decision maker's preferences.
8. Tell PASS to evaluate alternatives by clicking on Eval from the PASS window menu.

Fig. 4. Hierarchy of the major PASS modules.

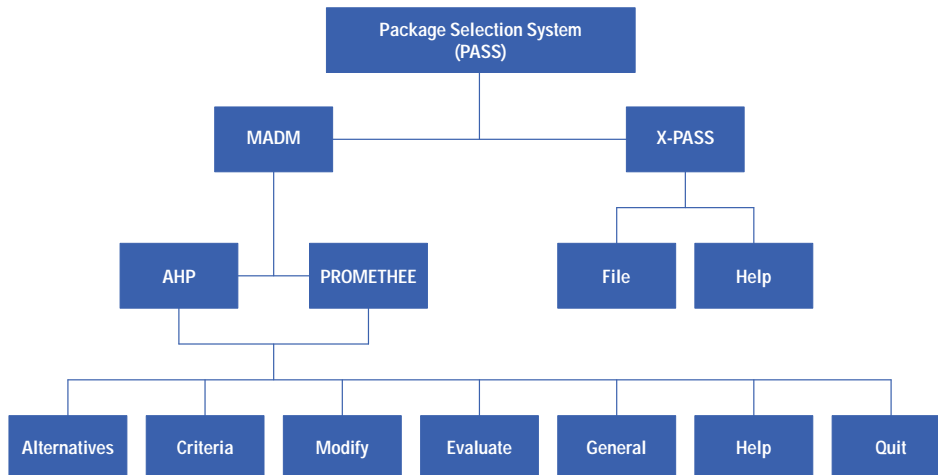
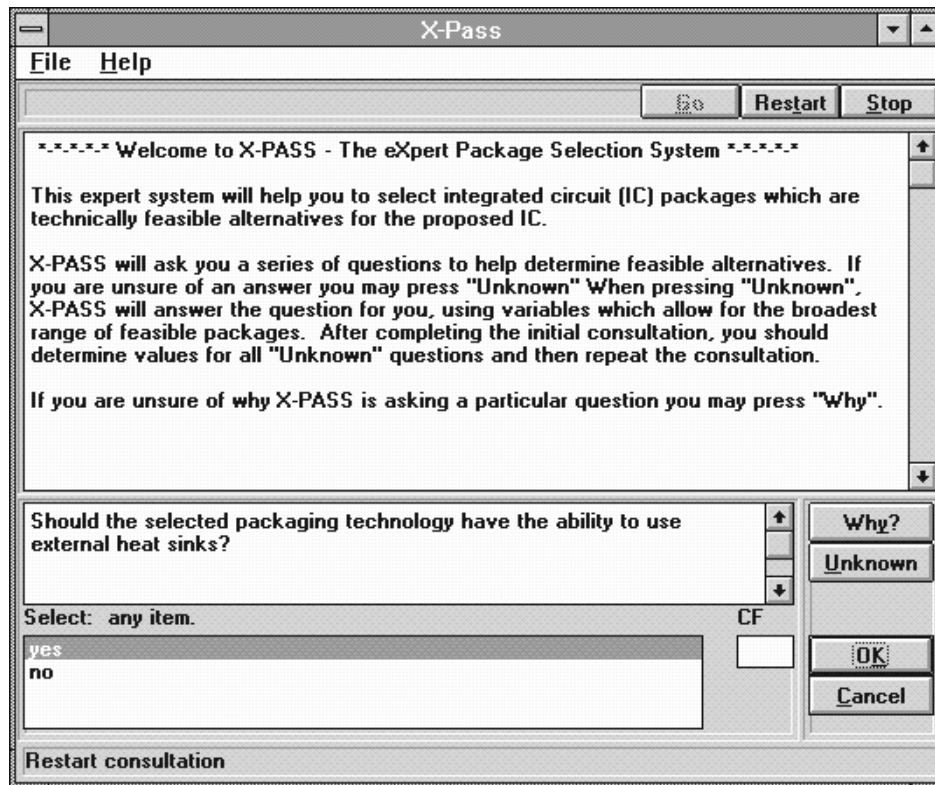


Fig. 5. X-PASS form.



PASS contains 18 separate modules for accomplishing this task. The modules are used for inputting data, selecting criteria and alternatives, creating custom alternatives, displaying results, and interfacing with other Windows applications. The hierarchy of the major PASS modules is shown in Fig. 4.

PASS can be started by clicking on the PASS icon or by typing in the proper path and `Pass.exe` in one of the Windows Run screens. During startup, a welcome screen is displayed. After a brief pause, a module selection screen is displayed. From this screen the user can choose an expert system consultation or select multiple-attribute decision making.

Selection of X-PASS causes the expert system module to start. The knowledge base for X-PASS is automatically loaded. The X-PASS screen is shown in Fig. 5. This form contains four control buttons: Go, Restart, Stop, and Cancel. The X-PASS screen also contains three response buttons: Why, Unknown, and OK.

Go is used to begin a consultation. Restart continues a consultation that was inadvertently aborted. Stop ends a consultation but preserves the current knowledge cache. Cancel ends the consultation immediately and clears the knowledge cache.

A user can respond to a question asked by X-PASS with Why, Unknown, or OK. A response of Why causes a message to appear that explains why the expert system needs that particular piece of information. Selecting Unknown forces the system to pick an alternative that is the least restrictive in terms of eliminating alternatives. It also associates a low certainty factor with that choice (a certainty factor is a number between 0 and 1 representing the decision maker's confidence in an answer given in response to a question from X-PASS). Selecting OK enters the user's highlighted response into the knowledge cache.

Also on the X-PASS form, certainty factors can be entered in the box labeled CF, and other knowledge bases can be entered into X-PASS through the use of the file menu. This powerful feature allows the X-PASS database to be modified by any user. It also allows X-PASS to be used as an expert system shell for other, nonpackaging-related problems. Feasible alternatives that are determined by a consultation with X-PASS are saved in an ASCII file for optional use by the MADM modules.

Clicking the MADM button on the module selection form initiates the process for using the multiple-attribute decision making tools. The first form in the MADM module (Fig. 6) allows the user to input the number of alternatives and criteria. The AHP or PROMETHEE technique is also selected by using this form. A list box on the form shows the decision maker all of the standard criteria supported by PASS. The user also has the option to check the box labeled Use X-PASS Alternatives. Selecting this option enters the feasible alternatives determined by the most recent X-PASS consultation into working memory. Otherwise, the decision maker can type the number of alternatives and criteria into the text boxes at the righthand side of the form. A maximum of 20 alternatives and 20 criteria can be used. This limitation is a result of the way in which Visual BASIC handles large arrays.

The remainder of the MADM forms are activated by using the menu on the main PASS form. Most of the data entry for PASS is self-explanatory and a help facility is provided to assist in data entry.

Fig. 6. Initial MADM form.

The screenshot shows a software window titled "Options". At the top, there are three radio buttons: "Promethee", "AHP", and "Use X-PASS Alternatives". Below this is a section titled "Possible Criteria" containing a list box with the following items: "Price (\$)", "Design (Wks)", "Lead Time (Wks)", "Overhead (hrs)", "Sourcing (#)", "Quality", "Rework", and "Prototyping". To the right of the list box are two input fields: "Alternatives" and "Criteria". Below these fields is a button with a checkmark icon labeled "Enter".

Entering Criteria for AHP and PROMETHEE

Criteria for AHP and PROMETHEE are entered in different ways. The criteria input modules are activated by selecting Crit from the main menu. For AHP, two forms are used to enter criteria. The first form (Fig. 7) is for alternatives selection and contains a combination list box labeled Criteria. As the name implies, it is a list of the standard criteria supported by the packaging selection system. After all criteria are entered into working memory, a second screen is activated. This screen, shown in Fig. 8, allows pairwise comparison of the criteria, based on Saaty's scale of measurement.

Comparisons are made by entering a number from Saaty's scale in the matrix in the upper lefthand corner. This causes a row to be compared with a column. The reciprocal comparison is automatically entered into the proper cell. After all comparisons have been made the weights and consistency index can be displayed by using the Eval button.

A consistency index greater than 0.1 indicates that modifications to the matrix may need to be made. The Reset button clears the weights and consistency index but leaves the matrix intact. This is useful if only slight modifications to the matrix are desired. The Clear button clears the matrix as well as the weights and consistency index. Pressing Exit enters all data into working memory and returns the system to the main menu. Saaty's scale is shown in a list box at the lower lefthand corner of the form.

Entering criteria for the PROMETHEE technique is a three-step process. The PROMETHEE form is shown in Fig. 9.

The first step is selecting a criterion from the list box in the criteria section of the form. The criterion is entered into working memory by pressing OK. A range of values for each of the criteria corresponding to the preselected alternatives is shown in the range list box.

The second step involves selecting a preference function by pressing one of the six buttons located in the upper lefthand corner of the form. When a preference function is selected, a graph of the function showing the parameters appears in the screen to the right of the buttons. For the form shown in Fig. 9, preference function four (P4) has been selected.

The third and final step is to enter the appropriate values for the parameter(s) in the text boxes located in the parameters section of the form. The listing of the range of data is useful for this step. All data is entered into working memory by clicking on Record. This process is repeated for all selected criteria.

Fig. 7. Selecting alternatives in AHP.

The screenshot shows a software window titled "Criteria". On the left, there is a text box containing "From the 'Criteria Combo Box', Select Criteria Number:" and a small input field with the number "1". To the right of this is a dropdown menu labeled "Criteria" with a downward arrow. The dropdown menu is open, showing a list of criteria: "Price (\$)", "Design (Wks)", "Lead Time (Wks)", "Overhead (hrs)", "Sourcing (#)", "Quality", "Rework", and "Prototyping". To the right of the dropdown menu is a button with a checkmark icon labeled "Enter".

Fig. 8. Making pairwise comparisons in AHP.

	C1	C2	C3	C4	C5
C1 = Price (\$)	1.00				
C2 = Design (Wks)		1.00			
C3 = Quality			1.00		
C4 = Lead Time				1.00	
C5 = Rework					1.00

Enter

Weights and Consistency Index

	C1	C2	C3	C4	C5
Weights					

Consistency Index

Saaty's Scale

- 1 - Equally important or preferred
- 3 - Slightly more important or preferred
- 5 - Strongly more important or preferred
- 7 - Very strongly more important or preferred

AHP

Controls

Eval

Reset

EXIT

Exit

Clear

Fig. 9. Entering criteria in PROMETHEE.

Enter the value for q in 'Parameter 1'. Enter the value for p in 'Parameter 2'. Enter the weight for criteria 1, then press 'Record Criteria'. Weights will be normalized over all criteria.

Parameters		Criteria	
Parameter 1	2	Price (\$)	RANGE
Parameter 2	6		14.75
Weight	1		18.80
			12.20
			9.25

Record

OK

PASS Output

Output from PASS is presented visually and interactively while using the software. ASCII files are also generated; these can be imported into reports or other applications. The output from X-PASS is a list of technically feasible alternatives. With each alternative a confidence factor is reported. The confidence factor ranges from 20% to 100% and is based on the number of questions answered as "unknown." The output from the MADM modules consists of a ranking of the alternatives and their associated weights. A sample of the ASCII output from an AHP session is shown in Fig. 10.

Fig. 10. ASCII output from AHP.

Number of Criteria is	5					
Number of Alternatives is	5					
CRITERIA	MIN/MAX					
C1 = Price (\$)	MIN					
C2 = Design (Wks)	MIN					
C3 = Lead Time (Wks)	MIN					
C4 = Quality	MAX					
C5 = Footprint	MIN					
Pairwise Comparisons						
CRITERIA	C1	C2	C3	C4	C5	
C1	1.000	3.000	9.000	5.000	7.000	
C2	0.333	1.000	6.000	2.000	3.000	
C3	0.111	0.167	1.000	0.333	0.500	
C4	0.200	0.500	3.000	1.000	2.000	
C5	0.143	0.333	2.000	0.500	1.000	
Consistency Index:	0.0119					
Alternatives						
A1 = 208-pcpga						
A2 = 208-cpga						
A3 = 208-cqfp						
A4 = 208-mquad						
A5 = 196-tab						
CRITERIA	A1	A2	A3	A4	A5	
Price (\$)	14.75	18.80	12.20	9.25	7.85	
Design (Wks)	8.00	6.00	8.00	12.00	4.00	
Lead Time (Wks)	6.00	1.00	1.00	4.00	5.50	
Quality	4.00	8.00	2.00	5.00	3.00	
Footprint	2.60	2.40	1.30	1.30	1.00	
Normalized Alternatives						
Note: Weights are computed from the principal right eigenvector of the pairwise comparisons matrix.						
CRITERIA	A1	A2	A3	A4	A5	Weight
Price (\$)	0.1546	0.1213	0.1869	0.2466	0.2905	0.5383
Design (Wks)	0.1667	0.2222	0.1667	0.1111	0.3333	0.2228
Lead Time (Wks)	0.0641	0.3848	0.3848	0.0962	0.0700	0.0430
Quality	0.1818	0.3636	0.0909	0.2273	0.1364	0.1222
Footprint	0.1152	0.1248	0.2303	0.2303	0.2994	0.0736
Ranking of the Alternatives						
Composite Weight						
Alternative 1 : 196-tab	0.2724					
Alternative 2 : 208-mquad	0.2064					
Alternative 3 : 208-cpga	0.1850					
Alternative 4 : 208-cqfp	0.1824					
Alternative 5 : 208-pcpga	0.1538					

Acknowledgments

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References

1. A.P. Sage, *Decision Support Systems Engineering*, John Wiley and Sons, Inc., 1991.
2. J.P. Ignizio, *Introduction to Expert Systems. The Development and Implementation of Rule-Based Expert Systems*, McGraw-Hill, 1991.
3. J.P. Brans and P.H. Vinke, "A Preference Ranking Organization Method (The PROMETHEE Method for Multiple Criteria Decision Making)," *Management Science*, Vol. 31, 1985, pp. 647-656.
4. J.P. Brans, P.H. Vinke, and B. Mareschal, "PROMETHEE: A New Family of Outranking Methods in Multicriteria Analysis," in J.P. Brans, ed., *Operations Research '84*, Elsevier Science Publishers, 1984, pp. 477-490.
5. T.L. Saaty, "Axiomatic Foundation of the Analytic Hierarchy Process," *Management Science*, Vol. 32, 1986, pp. 841-855.
6. P.T. Harker and L.G. Vargas, "Theory of Ratio Scale Estimation: Saaty's Analytic Hierarchy Process," *Management Science*, Vol. 33, 1987, pp. 1383-1403.
7. R. Saaty, "The Analytic Hierarchy Process—What it Is and How it Is Used," *Mathematical Modeling*, Vol. 9, 1987, pp. 161-176.
8. T.L. Saaty, *The Analytic Hierarchy Process*, McGraw-Hill, 1980.

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- ▶ [Go to Article 8](#)
 - ▶ [Go to Table of Contents](#)
 - ▶ [Go to HP Journal Home Page](#)