

Does Principal-Agent Theory Work in Real Life?

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ABSTRACT

We study the agency problem experimentally focusing on two issues that are central to its effectiveness. The first tests whether an incentive compatible direct revelation mechanism performs well when human agents are asked to report probabilistic information. The second addresses the principal's lack of knowledge as to how types and effort levels relate to the final outcome. Our results reveal several behavioral effects that reduce the efficiency of the principal-agent mechanism. We find out that human agents underestimate low probabilities and overestimate high probabilities, introducing errors into what should be a truth-telling mechanism. Furthermore, principals were observed to underpay their agents by substantial amounts. These behavioral issues may explain why contracts designed through standard principal-agent models are seldom used in practice.

1. Introduction

A number of problems within the enterprise are characterized by interactions between managers and employers in which there is asymmetry of information. This is part of the more general agency problem, in which one group is unable to fully monitor the efforts of another and so needs to align as much as possible the incentives of both groups. A whole body of work, principal-agent theory, has evolved in order to deal with these problems. This theory examines relationships in which one party -the principal - determines the work, which another party - the agent - undertakes. The theory argues that under conditions of incomplete information and uncertainty, which characterize most business settings, two agency problems arise: adverse selection and moral hazard. Adverse selection is the condition under which the principal cannot ascertain if the agent accurately represents his ability to do the work for which he is being paid. And moral hazard is the situation under which the principal cannot be sure if the agent exerts maximal effort.

An interesting and important example of such asymmetry of information within enterprises operating in dynamic environments is the accurate forecast of future sales. These forecasts not only affect the financial performance of a company but also the planning of production and the supply chain, as well as the advance purchase of parts that will be needed.

While a small part of the problem of forecasting lies in the quality of the data at hand, a bigger one is posed by the behavior of those individuals responsible for the sales. Although sales people have crucial private information with which to assess the probability of a given sale, simply asking them for this probability is likely to result in biased and idiosyncratic information. This is because a sales person under a quota system, in which the commission rate increases after total sales pass a certain level, has as primary incentive to complete deals. If asked also to provide probabilistic assessment of deals, his best strategy is to lowball his assessment hoping for a low

quota in the future. This is what is colloquially referred to as “sandbagging” and it constitutes a very common problem in business organizations.

The theoretical literature of principal-agent modeling (Laffont and Martimort (2002), Bolton and Dewatripont (2005)) has addressed theoretically both the issue of truthful reporting as well as the proper incentives needed to induce optimal effort levels in many environments. The literature typically assumes that the principal is unaware of the type and effort of the agents, while knowing the functional form of the outcome as a function of the agents’ type and effort level. This assumption, which is needed to derive the optimal schedule of payments is not realistic, for in most practical situations there may not be enough data for an outside expert to estimate these functions. Furthermore, it is assumed that agents maximize expected utility while principals design contracts according to that assumption.

However, human behavior does not always match theoretical modeling because such models often assume unrealistic levels of rationality on the part of its players (Camerer et al 1995, Kagel & Roth 1995). In this case, the problem is exacerbated by the fact that the principal has more limited information (i.e. the relationship between types, effort levels and outcomes) about the environment.

There are few studies that test the main predictions of agency theory in the laboratory. In one of the early examples, Berg et al. (1992) showed that the predictions of the theory under moral hazard hold experimentally most of the time when principal’s strategy space is limited and agent cannot reject the contract offer. Epstein (1992) observed that the results are less robust than those of Berg et al. (1992) when an explicit reservation wage for the agent is included. Keser and Willinger (2000) considered a principal-agent relationship with moral hazard and identified three basic principles, i.e., appropriateness, loss avoidance and sharing power, for principals to design their payment schemes. They observe that when using these principles, net expected surplus is more evenly allocated between the principal and the agent than the agency theory predicts.

However, in their follow-up paper, Keser and Willinger (2002) also show that when the effort costs are high, agency theory can do better than the prediction using the three principles. Other notable studies in this area include Anderhub et al (2002) and Cabrales and Charness (2003)¹. In the sales force compensation context, we are only aware of the experimental study by Ghosh and John (2000). They studied the agency problem by focusing on the effort selection issue, and found out that for settings in which risk neutral principals deal with risk averse agents whose actions are non verifiable, higher levels of effort-output uncertainty evoke more salary-weighted compensation plans, in agreement with the basic premises of agency theory.

Our study differs from the aforementioned papers in some aspects: First of all, we consider a combination of adverse selection and moral hazard by including both an independent market parameter drawn from a continuum and observed only by the agent, as well as her unverifiable effort. We also explore the case of a principal with limited information about the environment which has not, to the best of our knowledge, considered before.

We study the agency problem experimentally focusing on two issues in the context of sales forecasting. The first tests whether an incentive compatible direct revelation mechanism performs well when human agents are asked to report probabilistic information. The mechanism is based on a schedule of payments that are offered to the sales force and structured in such a way so as to trade off the fix portion of a sales representative's salary with the portion that is at risk. Furthermore, the sum of the fixed part and the bonus increases with the at-risks portion. Thus, for deals that are likely, a sales person is expected to choose a larger percentage of his salary at risk in hopes to gain a larger total. For deals that are not likely, she is expected to settle for a smaller at-risk portion. These choices reveal their assessment of probability of the deals.

¹ For a review of experiments on moral hazard and incentives, we refer the reader to Keser and Willinger (2002b).

Secondly, we address the issue of the principal's lack of knowledge of how types and effort levels relate to the final outcome. A fraction of the subjects are asked to play the role of the principal and to choose compensation schedules for their agents restricted to a class of direct mechanisms. The goal is to see if they can discover a compensation schedule that optimizes effort. In addition, we include treatments of more traditional compensation schemes to gauge the potential improvement over common business practice.

Our results reveal several behavioral effects that reduce the efficiency of the mechanism. In particular, human agents underestimate low probabilities and overestimate high probabilities, introducing errors into what should be a truth-telling mechanism. Furthermore, principals were observed to underpay their agents by substantial amounts. These behavioral issues may explain why contract designed by standard principal-agent modeling are seldom used in practice. We hope that the experimental data may shed some light on how the standard mechanism may be tweaked to produce better results.

The next section presents the models underlying our experiments. Experimental design is explained in Section 3. Section 4 reports the results, and Section 5 concludes.

2. Mechanism Design

In our single-principal-single-agent scenario, the sales agent works on a sales deal that results in a predetermined amount of revenue for the principal if it is successful. Probability of success depends on the effort of the agent and an exogenous random variable observed only by the agent and referred to as the market signal. The market signal is equivalent to the type of the sales agent in the standard principal-agent literature.

The sequence of events is as follows:

- The principal offers the agent a compensation menu.
- The agent observes the market signal and the compensation menu.
- The agent chooses his effort level and a contract from the menu.

- The deal is revealed to be successful or not. Depending on the outcome, the principal and the agent receive their payoffs.

We solve a standard principal-agent model with specific functional forms as the basis of the experimental design. We then conducted experiments in which subjects played the role of the principal, without any knowledge of the functional form of probability of success. The experiments taught us whether subjects can learn to find the optimal contract over time, a finding we describe in future section.

2.1 Optimal Mechanism with Knowledge of Success Probability

In this section, we describe the optimal mechanism based on standard principal-agent modeling, assuming that the principal knows how the success probability depends on the effort and the market signal. The probability of a sales deal, p , is given by $p = \min(\alpha e + \beta s, 1)$, where e denotes the agent's effort, s is the market signal and α and β are parameters. s is uniformly distributed between 0 and 1. The cost of effort is given by $C(e) = e^2/2$. The goal of these specific functions is to provide a concrete example for experimental implementation. While we do not claim that these are the only reasonable functions to be used for experiments, any reasonable probability function which increases in effort and market signal, and any reasonable convex cost of effort function which increases in effort will produce similar results.

The principal offers the agent a menu of contracts in the form $\{ x(t), y(t) \}$ where $x(t)$ is the fixed payment and $y(t)$ is the variable payment with $t \in [0,1]$. The agent receives the fixed payment independent of the success of the sales deal, but receives the variable payment only when the deal is successful. The agent chooses his contract by setting a value for the parameter t , which we call the agent's report.

Using the Extended Revelation Principle (Laffont and Martimort (2002)), we can restrict our attention to the class of truth-telling contracts in order to find the optimal menu of contracts for the principal. In this case, we can write the principal's problem as

$$\max_{x_0, y_0} E_s [R(\min(\alpha e(s, s) + \beta s, 1)) - x(s) - y(s)(\min(\alpha e(s, s) + \beta s, 1))]$$

$$x(s) + y(s)(\min(\alpha e(s, s) + \beta s, 1)) - \frac{e(s, s)^2}{2} \geq u \quad (1)$$

$$x(s) + y(s)(\min(\alpha e(s, s) + \beta s, 1)) - \frac{e(s, s)^2}{2} \geq x(\hat{s}) + y(\hat{s})(\min(\alpha e(\hat{s}, s) + \beta s, 1)) - \frac{e(\hat{s}, s)^2}{2} \quad (2)$$

where R is the revenue from the sales deal. In this program, $e(\hat{s}, s)$ is the optimal effort given that the agent observes the signal s and chooses the report \hat{s} . The first set of constraints pertains to the individual rationality (IR) constraints and they ensure that any type of the agent makes at least as much as his reservation profit, u , by participating in the contract. The second set of constraints is the incentive-compatibility (IC) constraints and they reflect the fact that the agent observing market a signal s has the option of choosing \hat{s} as a report but prefers to choose s .

The following proposition characterizes the payment functions found as a solution to the principal's problem.

Proposition 1: When the principal's objective is maximized, fixed and variable payment functions are given by

$$y(s) = (R\alpha^2 + \beta s - \beta) / (\alpha^2)$$

$$x(s) = x_s^0 - (\beta R - \beta^2 / \alpha^2) s - \beta^2 s^2 / \alpha^2$$

where x_s^0 is the fixed payment when the market signal is 0. •

Note that the fixed payment is a quadratic and decreasing function of the market signal while the variable payment is linearly increasing. The optimal effort is also an increasing function of s .

Since we test whether we obtain the true probability from the sales agent through her report, we use payment functions defined over probability in our experiments. We obtain these

functions by mapping the optimal payment functions characterized in Proposition 1 from market signal to probability. Given the market signal and given that the agent chooses the optimal effort, the probability of a successful deal is

$$p^*(s) = (R\alpha^2 + 2\beta s - \beta)$$

It is a one-to-one function since it is linearly increasing in s . Therefore, we can write s in terms of p^* as:

$$s = (p^*(s) - R\alpha^2 + \beta)/(2\beta)$$

Plugging the expression above into $x(s)$ and $y(s)$, we obtain the optimal payment functions in terms of p^* .

$$\hat{y}(p^*) = (R\alpha^2 + p^* - \beta)/(2\alpha^2)$$

$$\hat{x}(p^*) = x_s^0 - (R\alpha^2 - \beta)(p^* - R\alpha^2 + \beta)/(2\alpha^2) - (p^* - R\alpha^2 + \beta)^2/(4\alpha^2)$$

Figure 1 provides an example of the optimal payment functions when $\alpha=0.02$, $\beta=0.2$, $R=2000$ and $x_s^0=400$. For example, when success probability for the optimal effort is 0.5, the fixed payment is 469 and variable payment is 1375. However, when the probability is 0.6, the fixed payment decreases down to 400 and the variable payment becomes 1500. This means that while the probability of having a successful sales deal increases, agent's fixed payment is reduced and her variable payment is increased. Increases in the variable payment compensate decreases in the fixed payment as p^* (or equivalently, s) approaches to 1 and make their total an increasing function.

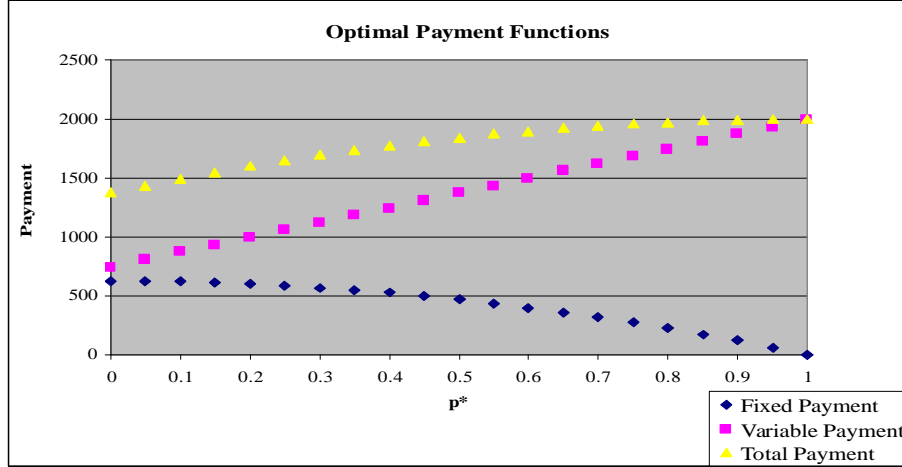


Figure 1 Optimal Payment Structure ($\sigma=0.02$, $\beta=0.2$, $R=2000$, $x_s^0=400$)

2.2 Mechanism Design without Knowledge of the Success Probability

In the previous section, we assumed that although the principal does not observe the market signal and the agent's choice of effort, he knows how they relate to the success probability. But in most environments, the principal may not know precisely how market signal and/or the sales effort affect the probability of success. Even if this is the case, we show in this section that we can find a menu of contracts for the principal to obtain truthful information from the sales agents.

The formulation of the previous section is based on the truthful revelation of the market signal. An equivalent formulation can be found where the agent is given an incentive to report the probability of success. If $\tilde{x}(t)$ and $\tilde{y}(t)$, the fixed and the variable payments in this setting, are assumed to be differentiable and p denotes the probability of success, the incentive compatibility constraint is given by

$$\frac{d\tilde{x}(t)}{dt} \Big|_{t=p} + p \frac{d\tilde{y}(t)}{dt} \Big|_{t=p} = 0 \tag{3}$$

Note that the contract menu is independent of the effort and the market signal once p is given. Therefore, any menu of contracts that satisfies the condition above and that provides the agent at least her reservation profit induces truth telling. Particularly, if we assume that $\tilde{x}(t)$ is a decreasing second-order quadratic function and $\tilde{y}(t)$ is linearly increasing (as in the previous section), using Equation (3) we find that

$$\tilde{x}(t) = c - \frac{bt^2}{2} \tag{4}$$

$$\tilde{y}(t) = bt \tag{5}$$

where b and c are constants. Since the objective of the agent is concave with respect to t , Equation (3) is also sufficient. A detailed analysis of the equilibria under $\tilde{x}(t)$ and $\tilde{y}(t)$ is given in Proposition 3 of the Appendix.

Our main motivation to use in our experiments the above functional forms was to analyze how they perform in terms of truth telling when they are presented to human subjects. Another reason is to find out whether the principals can offer relatively “good” contracts to their agents that will lead to higher efficiency and better information on the success probability of the sales, as deals compared to traditional methods. Therefore, the principal is asked to choose the total payment $P(t) = \tilde{x}(t) + \tilde{y}(t)$ at $t=0$ and $t=1$ in our experiments. In order to simplify the principal’s decision, the contract is then automatically filled in by the following equations

$$\tilde{x}(t) = P(0) - (P(1) - P(0))t^2$$

$$\tilde{y}(t) = 2(P(1) - P(0))t$$

which is consistent with Equations (4) and (5).

It is beyond the scope of this paper to provide a theoretical model of how a rational strategic principal will choose the contract since it involves modeling the principal’s beliefs about

the functional form, which can be very difficult. Instead we empirically test this approach with human subjects.

2.3 Benchmark Model

We use a model that mimics a simplified version of a typical business scenario to serve as the benchmark. In this scenario, the principal offers the agent a fixed payment that he will receive regardless of his actions, plus a bonus that will be paid only in the case of success. The key difference from the previous model is that both the fixed payment and the bonus are constant (i.e., the principal doesn't provide the agent with a menu). In this setting, the principal minimizes the function

$$\max_{x,y} E_s [R \min(\alpha e_s + \beta s, 1) - x - y \min(\alpha e_s + \beta s, 1)]$$

Where e_s is a solution to the agent's profit maximization problem that is given by

$$\max_e \{x + y \min(\alpha e + \beta s, 1) - e^2 / 2\}$$

The bonus y is determined from solving both the agent's and the principal's problem, whereas the fixed payment, x , is found from the least profit that the principal has to provide to the agent.

Proposition 2: The optimal bonus is given by $y = (2R\alpha^2 - \beta) / (4\alpha^2)$

If the amount of money paid to the agent (at a market signal of zero) in the principal-agent model and in the benchmark model is set equal to each other, the fixed payment in the latter is found by:

$$x = x_s^0 - (2R\alpha^2 - \beta)^2 / (32\alpha^2) + (R\alpha^2 - \beta)^2 / (2\alpha^2)$$

where x_s^0 is the fixed payment at $s=0$.

We can now compare the solution of the benchmark model and the principal-agent model in terms of the agent's and the principal's expected profit. It is expected that when the principal

offers a menu of contracts, he will earn higher profits due to higher flexibility. If the optimal y and x are such that

$$y = (2R\alpha^2 - \beta)/(4\alpha^2)$$

$$x = x_s^0 - (2R\alpha^2 - \beta)^2/(32\alpha^2) + (R\alpha^2 - \beta)^2/(2\alpha^2)$$

The difference in the principal's expected profit between the original and the benchmark model is given by:

$$\Delta_p = (R\alpha^2 - \beta)R/8 + (7/96)\beta^2/\alpha^2$$

The difference in the agent's expected profit between the original and the benchmark model is

$$\Delta_a = \beta(6R\alpha^2 - 5\beta)/(24\alpha^2)$$

Note that \bullet_p and \bullet_a are always positive once the conditions on the probabilities

$$p^*(1) = (R\alpha^2 + \beta) \leq 1$$

$$p^*(0) = (R\alpha^2 - \beta) \geq 0$$

are satisfied.

The differences in the expected profits indicate that both the agent and the principal are better off under the proposed compensation mechanism. Furthermore, this mechanism has the property of retrieving truthful information from the agent.

3. Experimental Design

The central point to be settled by the experiments is whether this mechanism can elicit non-biased information even when humans may not behave exactly as economic theory prescribes. Another issue to be elucidated is that the compensation mechanism depends on the assumption that the principal knows the probability relation, a fact which is not often possible in real life.

We conducted four sets of experiments: In the first set, a sales agent responds to a software principal who always uses an optimal payment schedule. The second set comprised the full principal-agent scenario where both the principal and the agent are human subjects. The third set is the same as the full principal-agent scenario, but we also added the ability to have limited communication between principals and agents before the contract setting stage. Finally, we used a treatment where the contract consists only of a single, pre-determined fixed payment and bonus to serve as a benchmark.

The experiments were implemented in the HP Experimental Economics Software Platform, MUMS. The subjects (mostly students from Stanford University) played the role of principals and sales agents. Standard HP Labs experimental economics procedures were followed. The experimental model was implemented in the HP experimental economics software platform and the experiments were conducted at the HP experimental economics laboratory. Subjects were paid according to their performance in the experiments, most of them making in the range of \$50-\$150 for an afternoon session. Before the subjects came into the lab, they had to go through a set of web based instructions and then pass a quiz.

In Agent-only experiment, we informed the agent about the value of her market signal and then, she decided on her report and her effort at each period. The market signal is independently drawn from continuous uniform distribution between 0 and 1 at each period. The sales agent was given the same menu of contracts (fixed and variable payment functions) over time that is optimal for a specified set of parameters. Each agent was given two minutes to make his decision. At the end of a period, the outcome of the sales deal was determined by a random draw where the success probability is given by $\min(e+s, 1)$. Sales agents may face nonpositive fixed payments depending on the set of parameters used during the game.

In the Benchmark experiment, each period consisted of four stages: In the first stage of the experiment, the principal set the fixed payment. In the second stage, the agent observed the

fixed payment and the market signal and she reported a value to the principal. The principal was not informed about the market signal. The agent's report, though does not directly affect agent's payments as they did before, was added to this experiment as a communication channel between the principal and the agent. In the third stage, the principal determined the variable payment. In the final stage, the agent chose her sales effort. The effort could not be observed by the principal as well. Each stage lasted for one minute. Each principal was matched with three sales agents working on separate sales deals at the beginning and played with the same agents through the game.

In the Principal-agent experiment, the principal moved first and were asked to decide on the total payment (fixed + variable payment) for the agent at the extreme range (0 and 1) of the report. Fixed and variable payments for each possible report were then generated automatically to enable truth telling. Once the payment menu was provided and the market signal was drawn as before, the agent decided on the report and the effort. The principal could not see the market signal or the effort of the agent. All the players were given two minutes to make their decisions. Each principal was matched with the same set of agents for the whole experiment.

In two of our principal-agent experiments, each principal was matched with three sales agents working on separate sales deals as in the benchmark experiment. Each principal played the game with the same agents at each period. In the last principal-agent experiment, each principal was matched in the first run and was randomly rematched in the second run with a single agent.

Principal-agent experiment with communication was the same as the principal-agent experiments, except every five periods, the principals and agents were allowed to exchange messages in the form of a possible contract (i.e total payment at report 0 and 1 and effort values when the market signal is 0, 0.5 and 1).

For all of the experiments, once the sales deal is revealed to be successful or not, a sales agent's payoff is found by adding fixed payment and variable payment if the sales deal is

successful and subtracting the cost of effort. A principal's payoff is found by subtracting the total payment to the agent from principal's sales revenue (0 if the deal is not successful). After each period, the agents were given the following feedback information: Market signal at the last period, their effort, their report, fixed and variable payment (corresponding to the report chosen except for the benchmark experiments), an indicator showing the deal was successful or not, payoff from the last period and cumulative payoff in the game. The principals were given a summary of their payment decisions, their agent's report, an indicator showing the deal was successful or not, payoff from the last period and their cumulative payoff.

It is worth noting that in all of the experiments, a principal was matched with the *same* set of agents for multiple periods. While this obviously introduced repeated game effects, we felt is necessarily so as to provide the principals a better chance to learn to adjust the menu of contracts for "their" agents. Because of the Folk Theorem, the mechanism can be even more efficient than the theory has predicted. Thus, we stacked the deck in favor of the theory. The results reported below are even more shocking in light of this.

4. Results

A total of 10 experiments were conducted. The following table summarizes the experimental settings. The first experiment was a pilot which provided a test of the software and procedures.

Table 1 Experiment overview

Experiment	Date	Type of the experiment	# of subjects	# of periods
1	08.05.2005	Pilot experiment	10	50
2	08.22.2005	Agent-only	9	50
3	09.02.2005	Principal-agent	12	25
4	09.06.2005	Principal-agent	12	25
5	09.08.2005	Benchmark	12	25
6	11.03.2005	Benchmark	12	30

7	07.07.2006	Principal-Agent w/Communication	16	10
8	07.21.2006	Principal-Agent w/Communication	12	25
9	08.25.2006	Principal-Agent (Single)	10	20 at each run (2 runs)

Result 1: The truth telling contract solicit at least as much information from the agents as in the benchmark contract. R-squared between the reported probabilities of success and the truth probabilities of success is used as the primary measure of information revelation. We calculated this measure pooling all of the observations from each experiment.

The following example illustrates why this measure was chosen. Consider an agent consistently reporting a^* (probability of success), where a is a constant between 0 and 1. If we use any measure based on absolute accuracy, we will obtain different results depending on a . If a is known however, the principal can back out a perfect forecast every time. Thus, this particular rule-of-thumb actually reveals all the information the agent has. The R-squared measure for this particular rule-of-thumb is always 100%, thus capturing all of the information that is revealed.

Table 2: R-squared of True Probability and Report

Experiment	Treatment	R-squared
1	Agent-only	0.61
2	Agent-only	0.87
3	Principal-Agent	0.69
4	Principal-Agent	0.80
5	PA with Communication	0.50
6	PA with Communication	0.40
7	Benchmark	0.30
8	Benchmark	0.62
9	Single PA	0.36

10	Single PA	0.29
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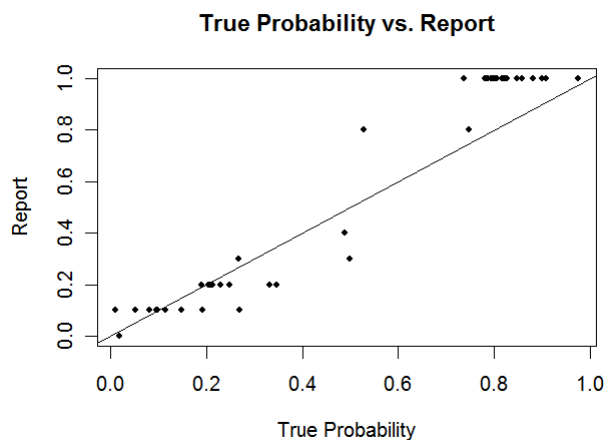
As one can see from table 2, out of the four agent-only and principal-agent experiments, three have higher R-squared than the two benchmark experiments. One out of those four has R-squared very close to one of the benchmark experiment and higher than the other one.

However, the principal-agent experiment with communication and the principal-agent experiment where single principal was matched with a single agent did not do so well with respect to information revelation. All of these four experiments have lower R-squared than the benchmark ones.

The surprising result is not so much as how the truth-telling contract performed, but that a fair amount of information was revealed in the benchmark experiments. Note that there is no incentive, in the one-period game, for agents to reveal any information in the benchmark experiment. Since the same principal was paired with the same agent in the experiments, reputation effect would encourage some information sharing. The surprise here is that properly designed truth-telling contract did not seem to have a significant advantage over the benchmark scenario when the game was repeated.

Result 2: Nonlinear behavior with respect to probability reporting was observed. In many instances, subjects were observed to under-report in low-probability ranges and over-report in very high-probability ranges. The following figure illustrates this phenomenon.

Figure 2 Example of the S-structure



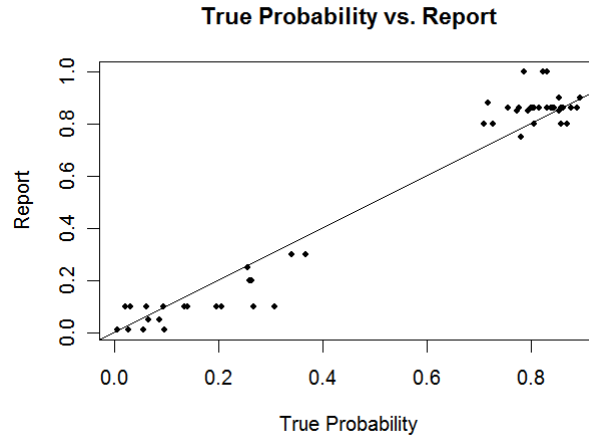
As can be seen, the data follows an S shape curve. One explanation of this behavior is that when the optimal report is low and there is a small to moderate chance of ending up with a successful deal, risk-averse subjects prefer to report low and gain from the fixed payment. On the other hand, when the probabilities of success is higher, subjects report high and try to take advantage of high variable payments. This means that risk considerations make the subjects deviate from maximizing their expected earnings.

To determine the extent of this behavior; we estimated a polynomial regression model with the report as the dependent variable and the truth probability as the independent variable. We found that in five out of ten experiments, the model is significant in the square or the cubic term of the polynomial, showing that the response is not linear at least in some of the experiments.

In addition, we found that clustering of the data points at the high and low end was quite common, and we observed this in the data for true probability vs. report as well. This can be understood from the subjects' relation with effort and probability: subjects choose low effort when the signal is small, which makes the probability of success small and report a low value as well. When the signal takes a moderate value, they choose a high effort that also increases the probability of success. As the signal becomes larger, even if they show a moderate effort, they

end up having a high probability value. The last two cases can be the explanation to the clustering at the high end while the first one explains the clustering in the low end.

Figure 3 Examples of Clustering



Result 3: The truth-telling contract caused significant under-reporting. In addition to nonlinear behavior, we found that subjects had a tendency to report probabilities lower than the truth. A paired Wilcoxon signed rank test was used to determine if the reported probabilities are LOWER than the true ones. The following table summarizes the results.

Table 3: P-values of Wilcoxon Signed Rank Test for alternative that Reported Probabilities < True Probabilities

Experiment	Treatment	P-value
1	Agent-only	<0.0001
2	Agent-only	<0.0001
3	Principal-Agent	<0.0001
4	Principal-Agent	<0.0001
5	PA with Communication	0.0221
6	PA with Communication	<0.0001
7	Benchmark	1.0000
8	Benchmark	1.0000
9	Single PA	0.0002

10	Single PA	0.0005
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As one can see, the test rejected the hypothesis, with very high significance, in favor of the alternative that the reported probabilities were LOWER than the true ones in all the experiments using the truth-telling contract. Worth noting that the null hypothesis that the reported probabilities were not smaller than the true ones cannot be rejected in both benchmark experiments. This is strong evidence that the contract was the cause of this lower-reporting phenomenon.

Result 4: The benchmark scenario resulted in significant over-reporting. We found that subjects had a tendency to report higher probabilities in the benchmark experiments. A paired Wilcoxon signed rank test was used to determine if the reported probabilities are HIGHER than the true ones. The following table summarizes the results.

Table 4: P-values of Wilcoxon Signed Rank Test for alternative that Reported Probabilities > True Probabilities

Experiment	Treatment	P-value
7	Benchmark	<0.0001
8	Benchmark	<0.0001

In the context of the period game, probability reporting in the benchmark experiments was cheap talk. Hence, there is no theoretical prediction. In the repeated game context, Folk theorem also does not provide clear unique predictions. Thus, we are left with little guidance from standard theory to interpret this result.

On the other hand, intuitively, this result is not difficult to understand. The agents had the incentive to paint a rosier picture in the hopes that the principals would increase their bonus. This explanation implies that the agents believe that principals would react to their reports naively.

Result 5: The truth-telling contract was NOT as efficient as the benchmark scenario. The following table summarizes the average payoff and the efficiency of each experiment.

Table 5: Payoffs and efficiencies

Experiment	Treatment	Average Payoff	Average Expected Payoff	Equilibrium Expected Payoff	Efficiency
1	Agent-only	370.34	361.48	410.59	0.88
2	Agent-only	15629.88	15790.08	17039.03	0.93
3	Principal-Agent	12202.25	12820.47	16863.46	0.76
4	Principal-Agent	11610.15	12501.93	16137.89	0.77
5	PA with Communication	11772.60	13755.27	17926.87	0.77
6	PA with Communication	11473.97	12578.08	17002.53	0.74
7	Benchmark	13023.41	13700.96	14118.33	0.97
8	Benchmark	16011.95	13370.56	14809.87	0.90
9	Single PA	15827.79	13792.56	17788.24	0.78
10	Single PA	12428.27	12065.76	16914.42	0.71

The efficiency was calculated based on average EXPECTED payoff. We used the expected payoff based on subjects' decisions, as opposed to using the actual realized payoff. Thus, this is a measurement of the value of their decisions.

As one can see, the two benchmark experiments have the higher efficiencies than all principal-agent experiments. The probability that the two benchmark experiments have higher efficiencies than all 6 principal-agent experiments, if ordering of the experiments based on efficiencies is random, is $(2! 6!)/8! \sim 3\%$.

The efficiencies of the two agent-only experiments are similar to that of the benchmark experiments. Thus, the low efficiencies in the principal-agent experiments were likely to be caused by the subjects' inability to offer the right contract.

Note that the value of the forecast was not factored into this analysis and we assume there are some exogenous reasons why accurate forecasts are desired.

Result 6: Communication did increase neither information revelation nor efficiency. As shown in the table above, the treatment where principal-agent communication was allowed did not result in a significant improvement in information revelation or efficiencies. Thus, it is unlikely that the low performance of the truth-telling contract was due to a lack of communication between subjects.

5. Conclusions and Future Research

Central to whether principal-agent mechanisms can be made practical, is whether the game theory model is robust with respect to the stringent rationality requirements imposed upon the decision makers. For example, we employed the solution concept of the Bayesian Nash equilibrium to predict how people behave in this mechanism. The solution requires the principal to have perfect knowledge of the probability relation of effort, market signal and the outcome (i.e. the deal is successful or not) and relevant utilities function for themselves, and then determine the best contract schedule by solving mathematical equations. The agents are assumed to be risk-neutral expected utility maximizers who also solve complex mathematical problems before arriving at the decisions. This is obviously beyond the undertaking of even the most sophisticated managers and sales agents. Furthermore, there is ample evidence to show that people are neither risk-neutral nor even adhering to expected utility maximization. The real question is whether theory is a good enough approximation so that the insight gained, mainly the use of menu of contracts to solve the problem of information asymmetry, is robust to behavioral effects.

In this paper, we explored behavioral issues with respect to the application of principal-agent type mechanisms to the sales forecasting problem. While human subject experiments showed that a properly designed mechanism can elicit reasonable amount of information, we observed substantial behavioral deviations from model predictions. In particular, we found that subjects tends to over-estimate high probabilities (>0.5) and under-estimate low ones. In addition, managers tended to underpay their agents and that resulted into lower levels of effort.

The research reported here provides answers along two dimensions that were not previously addressed. First, we expanded the scope of the agency problem to include soliciting truthful forecasts from the agents. Second, we solved this problem endogenously with respect to choosing the menu of contracts. While the true principals, i.e., the managers of the sales force, may not have either the data or the analytical skills to estimate the relevant functions and derive the subsequent optimal solutions, we created a mechanism that allows them to learn the solution through repeated interaction with the agents. Our results suggest that it is not enough to address the issue of principals' lack of knowledge of functional dependencies and that the mechanism design process needs to incorporate behavioral effects.

There are two possible future directions of this research. First, we are planning to redesign the mechanism with respect to behavioral issues. In addition, there are many additional variations in the environment, such as information discovery, multiple deals, soliciting a revenue forecast instead of a probability of success, and multiple agents working on the same deal. Additional research will be needed to develop a mechanism to adapt to these characteristics.

Another direction is to explore a field test within a real business to further investigate whether behavioral effects observed inside the lab will be consistent with real business environments. This obviously would be much further down the road since it is less likely a business is willing to subject itself to new processes unless there is some evidence that it will provide additional value.

Nevertheless we feel that this study elucidates a number of problems with principal agent theory which offer plausible reasons as to why contracts designed through standard principal-agent models are seldom used in practice.

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APPENDIX:

Note that $\alpha, \beta, R > 0$ in our models. The following lemma will be used when we identify the optimal solution to the principals’ problem:

Lemma 1: Equation 2 is satisfied if and only if the following conditions hold:

$$(C1) \quad \Phi(s, s) \equiv U(s) = U(0) + \int_0^s \beta y(\tau) d\tau$$

$$(C2) \quad y(s) \text{ is nondecreasing in } s$$

Proof of Lemma 1: We begin by showing that the IC constraints imply (C1) and (C2). We can rewrite the IC constraints as $\hat{s} \in \arg \max_s \{ \Phi(\hat{s}, s) - U(s) \}$ where

$$U(s) = \Phi(s, s) = x(s) + y(s)\beta s + \frac{\alpha^2 y(s)^2}{2}. \text{ This implies that}$$

$dU(s)/ds|_{s=\hat{s}} = \partial\Phi(\hat{s}, s)/\partial s|_{s=\hat{s}} = \beta y(\hat{s})$. Using the Fundamental Theorem of Calculus, we have

$$U(s) = U(0) + \int_0^s \beta y(\tau) d\tau$$

To see how IC constraints imply $y(s)$ to be nondecreasing in s , we compare the IC conditions for \hat{s} and s where $\hat{s} \leq s$:

$$x(\hat{s}) + y(\hat{s})\beta\hat{s} + \frac{\alpha^2 y(\hat{s})^2}{2} \geq x(s) + y(s)\beta s + \frac{\alpha^2 y(s)^2}{2}$$

$$x(s) + y(s)\beta s + \frac{\alpha^2 y(s)^2}{2} \geq x(\hat{s}) + y(\hat{s})\beta\hat{s} + \frac{\alpha^2 y(\hat{s})^2}{2}$$

They can be rewritten as:

$$y(\hat{s})\beta\hat{s} + \frac{\alpha^2 y(\hat{s})^2}{2} - y(s)\beta\hat{s} - \frac{\alpha^2 y(s)^2}{2} \geq x(s) - x(\hat{s}) \geq y(\hat{s})\beta s + \frac{\alpha^2 y(\hat{s})^2}{2} - y(s)\beta s - \frac{\alpha^2 y(s)^2}{2}$$

$$(y(\hat{s}) - y(s))\beta\hat{s} \geq (y(\hat{s}) - y(s))\beta s$$

Since $\hat{s} \leq s$, the inequality above implies that $y(\hat{s}) \leq y(s)$.

We will now show that (C1) and (C2) imply IC constraints. We define the net benefit from reporting \hat{s} when the agent observes s as

$$\phi(\hat{s}, s) = \Phi(\hat{s}, s) - U(s)$$

If both sides of the equation are differentiated with respect to s for a fixed \hat{s} , we get

$$\partial\phi(\hat{s}, s)/\partial s = \partial\Phi(\hat{s}, s)/\partial s - dU(s)/ds = \beta y(\hat{s}) - \beta y(s)$$

Note that if $\hat{s} < s$, $\partial\phi(\hat{s}, s)/\partial s \geq 0$, while if $s < \hat{s}$, $\partial\phi(\hat{s}, s)/\partial s \leq 0$. Therefore, this function is maximized when $s = \hat{s}$ and IC is satisfied.

Proof of Proposition 1: We can define $e(t, s)$ as:

$$e(t, s) \in \arg \max_e \{x(t) + y(t)(\alpha e + \beta s) - \frac{e^2}{2}\}$$

Note that we assume $e(t, s) + s$ does not exceed 1. We solve for the agent's best response and principal's maximization program under this assumption. We check whether the solution we find satisfies the assumption.

The first-order condition to find the optimal effort given the report t and signal s is given by:

$$y(t) * \alpha = e$$

Since the agent's expected profit given the report and the signal is concave in the effort, this first-order condition is also sufficient. Hence, the agent's maximum profit given that he observes s and reports t is found from:

$$\Phi(t, s) = x(t) + y(t)(\alpha^2 y(t) + \beta s) - \frac{\alpha^2 y(t)^2}{2} = x(t) + y(t)\beta s + \frac{\alpha^2 y(t)^2}{2}$$

The expected payments from the principal to an agent of type s are

$$x(s) + y(s)(\alpha^2 y(s) + \beta s) = U(s) + \frac{\alpha^2 y(s)^2}{2} = U(0) + \int_0^s \beta y(\tau) d\tau + \frac{\alpha^2 y(s)^2}{2}$$

Since the payments can be increased and the objective can be improved when $U(0) > u$, in the optimal solution it must be the case that $U(0) = u$. Using the relation above, we can write the principal's objective as

$$\max_{y_0} \left\{ \int_0^1 (R(\alpha^2 y(s) + \beta s) - u - \int_0^s \beta y(\tau) d\tau - \frac{\alpha^2 y(s)^2}{2}) ds \right\}$$

s.t (C2)

We can omit u for optimization purposes. The objective can then be rewritten using integration by parts:

$$\max_{y_0} \left\{ \int_0^1 (R(\alpha^2 y(s) + \beta s) - (s-1)\beta y(s) - \frac{\alpha^2 y(s)^2}{2}) ds \right\}$$

We first find the solution to the problem by relaxing the constraint (C2) and later check if that condition is satisfied. Optimal solution to the relaxed problem found by pointwise maximization is given by

$$y(s) = (R\alpha^2 + \beta s - \beta)/(\alpha^2)$$

Since this is a nondecreasing function of s , (C2) is satisfied and the solution is optimal for the principal's problem. Using this function and the agent's expected profit $U(s)$, $x(s)$ is given as

$$x(s) = x_s^0 - (\beta R - \beta^2 / \alpha^2)s - \beta^2 s^2 / \alpha^2$$

where x_s^0 is the fixed payment part of the contract when the report (or similarly, the market signal) is 0. We will use x_s^0 as a parameter in our experiments instead of the reservation profit u for simplicity.

Under $y(s)$ and $x(s)$ found here, agent's profit function is jointly concave with respect to t and e . Hence, first-order conditions are sufficient once $\bullet e(s,s) + \bullet s$ does not exceed 1 and $t \in [0,1]$. $\bullet e(s,s) + \bullet s$ does not exceed 1 for all s for the parameter sets we used in our experiments ($\alpha = 0.00353553, \beta = 0.5, R = 40000$ in all experiments except for the pilot agent-only experiment, $\alpha = 0.023, \beta = 0.4, R = 1000$ for the pilot experiment). Hence, the equilibrium we find is valid. Since we use the solution found here in agent-only experiments and provide the subjects with the menu to which they have a unique best-response (under expected profit maximization), we omit further discussion on other possible equilibria. •

Proof of Proposition 2: We first fully characterize the equilibria under the benchmark model.

$$\text{Agent's objective is } \max_e \{x + y \min(\alpha e + \beta s, 1) - e^2 / 2\}$$

e will not be set such that $e > (1 - \beta s) / \alpha$ since the effort is costly and additional effort will not increase the probability. Therefore, we can rewrite the problem as

$$\pi_a = \max_{0 \leq e \leq (1 - \beta s) / \alpha} \{x + y \min(\alpha e + \beta s, 1) - e^2 / 2\}$$

First-order conditions are given by

$$\partial \pi_a / \partial e = \alpha y - e = 0$$

$$\partial^2 \pi_a / \partial e^2 = -1 < 0$$

Note that the objective is concave in e . Optimal effort is found to be

$$e^*(s, y) = \min(\alpha y, (1 - \beta s) / \alpha)$$

Principal's objective can be written as

$$\max_{x, y} \int_0^1 \{(R - y)(\alpha e^*(s, y) + \beta s) - x\} ds$$

Define s_b as

$$\alpha y = (1 - \beta s_b) / \alpha \Rightarrow s_b = (1 - \alpha^2 y) / \beta$$

We consider three cases based on the value of s_b :

$$\text{Case 1: } s_b < 0 \Rightarrow e^*(s, y) = (1 - \beta s) / \alpha$$

In this case, principal's maximization problem turns into

$$\max_{x, y} (R - y - x) \text{ s.t. } (1 - \alpha^2 y) / \beta \leq 0 \Rightarrow y \geq 1 / (\alpha^2)$$

Since the objective is decreasing in y , y is chosen to be equal to $1 / (\alpha^2)$.

$$\text{Case 2: } s_b > 1 \Rightarrow e^*(s, y) = \alpha y$$

In this case, principal's maximization problem can be written as

$$\max_{x, y} \int_0^1 \{(R - y)(\alpha^2 y + \beta s) - x\} ds = (R - y)\alpha^2 y + 1/2(R - y)\beta - x \quad \text{s.t. } (1 - \alpha^2 y) / \beta \geq 1$$

First-order condition is given by

$$R\alpha^2 - 2y\alpha^2 - \beta / 2 = 0 \Rightarrow (2R\alpha^2 - \beta) / (4\alpha^2)$$

Since the second-order condition is negative, principal's objective is concave in y .

Taking the bounds on y into consideration, optimal y is found from

$$y = \min\{\max\{0, (2R\alpha^2 - \beta)/(4\alpha^2)\}, (1 - \beta)/(\alpha^2)\}$$

Case 3: $0 \leq s_b \leq 1$

In this case, principal's maximization problem can be written as

$$\begin{aligned} & \max_{x, y} \int_0^{s_b} \{(R - y)(\alpha^2 y + \beta s) - x\} ds + \int_{s_b}^1 (R - y - x) ds \\ & = (R - y)\alpha^2 y s_b + (R - y)\beta s_b^2 / 2 - x s_b + (R - y - x)(1 - s_b) \\ & \text{s.t} \\ & 0 \leq (1 - \alpha^2 y) / \beta \leq 1 \Rightarrow (1 - \beta) / \alpha^2 \leq y \leq 1 / \alpha^2 \end{aligned}$$

First-order condition is given by

$$-1 - (1 + (2R - 3y)\alpha^2)(-1 + y\alpha^2) / (2\beta) = 0$$

Since the first-order condition at $y = 1/\alpha^2$ is negative, this point can be eliminated. Therefore, the optimal solution in this case is found by comparing the objective values of $y = (1 - \beta)/\alpha^2$ (lower bound on y) and y we find from the first-order condition.

Optimal y to principal's problem is found by comparing the optimal solutions at each case.

For the parameter set $\alpha = 0.00353553$, $\beta = 0.5$, $R = 40000$ (which is the parameter set we used in our benchmark, principal-agent and principal-agent with communication experiments), Case 2 becomes optimal and $y = (2R\alpha^2 - \beta)/(4\alpha^2)$. Optimal effort of the agent is found by $e^* = \alpha y$.

Proof of Proposition 3:

Agent's problem can be written as

$$\begin{aligned} & \max_{t, e} \{c - bt^2 / 2 + bt \min(\alpha e + \beta s, 1) - e^2 / 2\} \\ & \text{s.t} \\ & 0 \leq t \leq 1 \\ & e \geq 0 \end{aligned}$$

e will not be set such that $e > (1 - \beta s) / \alpha$ since the effort is costly and additional effort will not increase the probability. Therefore, we can rewrite the problem as

$$\begin{aligned} & \max_{t, e} \{c - bt^2 / 2 + bt(\alpha e + \beta s) - e^2 / 2\} \\ & s.t \\ & 0 \leq t \leq 1 \\ & 0 \leq e \leq (1 - \beta s) / \alpha \end{aligned}$$

The objective is jointly concave in t and e when $0 \leq b \leq 1 / \alpha^2$. In this case, optimal t^* and e^* are given by

$$\begin{aligned} t^* &= \alpha e + \beta s \\ e^* &= \min\{b\alpha, (1 - \beta s) / \alpha\} \end{aligned}$$

The agent's best response is

$$\begin{aligned} e^* &= b\alpha\beta s / (1 - b\alpha^2) \text{ and } t^* = \alpha e^* + \beta s && \text{if } (b\beta\alpha s) / (1 - b\alpha^2) \leq (1 - \beta s) / \alpha, 0 \leq b \leq 1 / \alpha^2 \\ e^* &= (1 - \beta s) / \alpha \text{ and } t^* = 1 && \text{if } (b\beta\alpha s) / (1 - b\alpha^2) \geq (1 - \beta s) / \alpha, 0 \leq b \leq 1 / \alpha^2 \end{aligned}$$

The principal's problem can be written as

$$\max_c, b \left\{ \int_0^1 ((R - bt^*(s))(\alpha e^*(s) + \beta s) - c + bt^*(s)^2 / 2) ds \right\}$$

We can omit c for optimization purposes. We define s_d to be the point where $b\alpha\beta s / (1 - b\alpha^2) = (1 - \beta s) / \alpha$. ($s_d = (1 - b\alpha^2) / \beta$)

Based on the principal's objective, there are three cases:

$$\text{Case 1: } s_d < 0 \Rightarrow e^* = (1 - \beta s) / \alpha, t^* = 1$$

In this case, principal's maximization problem turns into

$$\max_b (R - b/2)$$

s.t

$$(1 - b\alpha^2)/\beta \leq 0 \Rightarrow b \geq 1/(\alpha^2)$$

Since the objective is decreasing in b , b is chosen to be equal to $1/(\alpha^2)$. Principal's expected profit is $R - 1/(2\alpha^2)$.

$$\text{Case 2: } s_d > 1 \Rightarrow e^* = b\beta\alpha s / (1 - b\alpha^2), t^* = \alpha e^* + \beta s$$

In this case, principal's maximization problem can be written as

$$\max_{x,y} \int_0^1 \{ (R - b(\alpha^2 b\beta s / (1 - b\alpha^2) + \beta s)) (\alpha^2 b\beta s / (1 - b\alpha^2) + \beta s) + b(\alpha^2 b\beta s / (1 - b\alpha^2) + \beta s)^2 / 2 \} ds$$

s.t

$$0 \leq b \leq (1 - \beta)/(\alpha^2)$$

First-order conditions of the principal's objective is given by

$$-(3R\alpha^2\beta + \beta^2)/(6(1 - b\alpha^2)^2) + (3R\alpha^2\beta b - 3R\beta + b\beta^2)\alpha^2 / (-1 + b\alpha^2)^3 = 0$$

The solution to this equation is $b = (3R\alpha^2 - \beta)/(\alpha^2(3R\alpha^2 + \beta))$.

Note that the derivative of the principal's objective at $b=0$ is positive when all of the parameters are positive. Therefore, this point is eliminated.

Objective value at $b = (3R\alpha^2 - \beta)/(\alpha^2(3R\alpha^2 + \beta))$ and $b = (1 - \beta)/(\alpha^2)$ must be compared if $(3R\alpha^2 - \beta)/(\alpha^2(3R\alpha^2 + \beta)) \leq (1 - \beta)/(\alpha^2)$. If the condition is not satisfied, the only candidate solution is $b = (1 - \beta)/(\alpha^2)$.

$$\text{Case 3: } 0 \leq s_d \leq 1 \Rightarrow (1 - \beta)/\alpha^2 \leq b \leq 1/\alpha^2$$

In this case, principal's maximization problem can be written as

$$\begin{aligned}
& \max_b \int_0^{s_d} \{ (R - b(\alpha^2 b \beta s / (1 - b\alpha^2) + \beta s)) (\alpha^2 b \beta s / (1 - b\alpha^2) + \beta s) + b(\alpha^2 b \beta s / (1 - b\alpha^2) + \beta s)^2 / 2 \} ds \\
& + \int_{s_d}^1 (R - b/2) ds \\
& \text{s.t} \\
& (1 - \beta) / \alpha^2 \leq b \leq 1 / \alpha^2
\end{aligned}$$

First-order conditions of the principal's objective is given by

$$-((2b - 3R)(-1 + b\alpha^2) + 3(b - 2R)\beta) / (6\beta) = 0$$

The solution to this equation is $b = (2 + 3(R\alpha^2 - \beta)) / (4\alpha^2)$.

Since the second-order condition is $-(2\alpha^2) / (3\beta) < 0$, the function is concave with respect to b .

Hence, once the solution found from the first-order conditions is in the bounds, it is optimal for this case. Otherwise, we need to compare the objective values at bounds $(1 - \beta) / (\alpha^2)$ and $1 / (\alpha^2)$.

For the parameter set $\alpha = 0.00353553, \beta = 0.5, R = 40000$ (which is the parameter set we used in our benchmark, principal-agent and principal-agent with communication experiments), Case 2 becomes optimal and $b = 40000$. •