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Quantum Solution of Coordination Problems

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We present a quantum solution to coordination problems that can be implemented with existing technologies. Using the properties of entangled states, this quantum mechanism allows participants to rapidly find suitable correlated choices as an alternative to conventional approaches relying on explicit communication, prior commitment or trusted third parties. Unlike prior proposals for quantum games our approach retains the same choices as in the classical game and instead utilizes quantum entanglement as an extra resource to aid the participants in their choices.

KEY WORDS: Coordination games; mechanism design; quantum information.

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1. COORDINATION GAMES

The existence of multiple equilibria in economic systems can lead to coordination failures and consequently to inefficient outcomes. Examples that have been extensively studied include firms deciding whether or not to enter a competitive market and how to position their offerings, and the coordinated resolution of social dilemmas involved in the provision of public goods.

Coordination problems have long been studied in the context of game theory,(5,9,19) where the coordination game is specified by a payoff matrix which yields several Nash equilibria. These equilibria can at times give the same payoff to all players, in which case the problem is for them to agree on which one to use, or different payoffs, leading to a competitive coordination game in which players prefer different equilibria. More generally these examples of games with multiple equilibria may require players

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to coordinate their choices to achieve a particular equilibrium.[2] Without such coordination the players can spend inordinate amounts of time trying to settle on an equilibrium, with consequent loss of the opportunity for high-payoff from coordinated choices.

1.1. Examples of Coordination Games

A simple example of a cooperative coordination game is that of two people having to choose whether to drive on the left or the right side of the road. Table 1 shows a payoff matrix for this game: a benefit if players make the same choices and a large penalty for different choices. Thus, this game has two Nash equilibria, with equal payoffs, corresponding to both drivers choosing the same side of the road. The coordination problem consists in both drivers finding a way to agree on which side of the road to drive.

Another cooperative coordination problem arises when using asynchronous exchanges, such as email, to coordinate a meeting location when one is not sure that recipients have read their emails and acknowledgements before the start of the meeting.[4,17] This illustrates how coordination games involve the issue of achieving common knowledge,[9] i.e., all parties know a particular choice to make and that the other participants also know this choice. For example, suppose two groups may be arranging separate meetings in one of two locations (of different desirability), but they do not know if the two meetings will be scheduled at the same time. If they are, as the first few people from each group show up at the more desirable location, they wish to flip a coin, as a fair way to decide which group gets the better location, and then make sure others in their group achieve common knowledge of the meeting location while limited to asynchronous communication. An interesting variation of this game is when only some members of each group may be interested in any particular meeting, and these members are not mutually known to each other prior to the time they actually appear at the meeting.

Table 1. Payoff Structure for Two Driving Choices: Left (L), Right (R). Each Row and Column Corresponds to Choices Made by the First and Second Players, Respectively, and their Corresponding Payoffs

Choice	L	R
L	1, 1	−3, −3
R	−3, −3	1, 1

Interesting competitive coordination games arise, for example, in the case of two players trying to coordinate on a mixed strategy against a third one without resorting to previous agreement or communication, as in the case of a coordinated attack on a rival or enemy. For instance, consider two military allies, on opposite sides of a field, who want to get a target held by an adversary on either the left or right sides of the field. The first of the allies can create a distraction. The other has the personnel and equipment needed to find the target provided they can do so undisturbed. The allies need to decide whether to send their forces to the left or right sides of the field. For any chance of success, the distraction from ally 1 must be on the opposite side of the field from where ally 2 goes. The payoff to the allies are shown in Table 2 (with the payoff to the adversary equal to 1 minus that of the allies). These payoffs express the fact that ally 2 must choose the same place as where the adversary hid the target (which will happen 1/2 the time in case the adversary uses a mixed strategy of each choice randomly selected with equal probability), but also be sure to do the opposite of ally 1. As a further aspect of this game, the allies wish to avoid communication and want to delay their decision as long as possible since the adversary could move the target if he knows which choice the allies make well in advance of their action, for example by spies or eavesdropping. This game has a mixed strategy equilibrium and the requirement for anti-correlation among the allies. Specifically, the adversary could select each choice with probability 1/2. If the allies similarly make random choices without coordination, their expected payoff is only 1/4. If instead they always coordinate their choices, i.e., never have both picking left or both picking right, their expected payoff increases to 1/2.

Table 2. Payoff Structure for the Two Allies and their Adversary Based on their Choices. For Example, the Entry of the Third Column, Second Row Corresponds to the First Ally going to the left, the second ally going to the right and the adversary hiding the target on the Left. The Payoff in this Case is 0 for the Allies and 1 for the Adversary

Allies' choices		Adversary's choice	
Ally 1	Ally 2	Left	Right
Left	Left	0	0
Left	Right	0	1
Right	Left	1	0
Right	Right	0	0

Table 3. Payoff Structure for the Rock, Paper, Scissors for the Pair of Allied Players Against the Third Player. Each Row and Column Corresponds to Choices made by the Pair (Assuming that they are the same) and Their Opponent, Respectively, and their Corresponding Payoffs. For Example, the Entry of the Second Column, Second Row Corresponds to the Allies both Choosing Paper and the Third Player Choosing Rock

Choice	Rock	Paper	Scissors
Rock	$0, 0$	$-1, 1$	$1, -1$
Paper	$1, -1$	$0, 0$	$-1, 1$
Scissors	$-1, 1$	$1, -1$	$0, 0$

Another example is a three-player version of the game of rock, paper, scissors, in which the two allied players must make the same choice to have any chance of winning. If the allies make different choices their payoffs are zero and the third player gets a payoff of 1. When the two allies make the same choice the payoff to the allies and the third player are given the payoff matrix of the usual two-player rock, paper scissors game, which is shown in Table 3.

This game has the feature that no single choice is best, i.e., there is no pure strategy Nash equilibrium. Instead, the best strategy for rational players is to make the choices randomly and with equal probability, which gives it a mixed strategy Nash equilibrium with expected payoff of 1/3.

For the full game without coordination the pair of allied players only has 1/3 chance of making the same choice, and another 1/3 to win against their opponent, leading to an expected payoff of 1/9. If they can be perfectly coordinated their payoff would be 1/3. In this example it is necessary to play random choices because any *a priori* commitment between the allied pair to a specific set of choices would no longer be a random strategy, and therefore discoverable by observation.

1.2. Solving Coordination Problems

These and many other instances of coordination problems can be solved in several ways. A first solution resorts to a trusted third party who knows the preferences of the participants and is given the authority to pick an equilibrium which is then broadcasted to the players. In the case of competitive coordination problems the trusted third party may also have enforcement powers, since some players may wish to move the group to another equilibrium with higher personal utility.

Another solution to coordination problems involves communication among players so that they can negotiate a choice. In the case of cooperative games even one player flipping a coin and broadcasting the result as the corresponding choice provides an effective solution. In a competitive setting, the negotiation might be such that the players wish to choose their equilibria at random as it would then be perceived as a fair choice. This case would require a trusted mechanism of coin flipping over a communication line, which can be enforced through cryptographic protocols.

A third mechanism for solving coordination problems invokes social norms, in which common knowledge of the participants' preferences can distinguish one equilibrium from the others, as in the case of choosing the largest river as a boundary between two countries. Such distinguished equilibria are often called focal points.[10,19]

While these mechanisms can solve coordination problems, there are times when none of these options are available, either because they are too expensive, slow or difficult to implement, or because privacy worries prevent the participants from using any of these options. Furthermore, a constraint from a larger context, such as the need to use a mixed strategy, might make it disadvantageous for players to have their choices revealed in advance. For instance, relying on communication could fail if jammed by an adversary, or used to identify the fact that players are communicating from specific locations—information that in itself could be damaging to reveal. In particular, if one player awaits an acknowledgement from the other, uncertainty over whether lack of response is due to jamming the response or lack of reception of the original communication could prevent establishing common knowledge needed to establish coordination.

This requirement for random but correlated choices without communication at the time the choices are made could be achieved by flipping a coin in advance, setting another coin to match the first one, hiding each coin in a separate box, and giving one box to each player. Opening a box at some later time gives an outcome correlated with that of the other player. A corresponding algorithmic approach would be prior agreement on a secret seed for use with a pseudorandom number generator. This amounts to prior agreement on the *method* for determining a choice, rather than the choice itself. Still, an adversary could learn this method of how the players will choose long before they actually do, use it to determine the choice to be made, and thus adjust its strategy accordingly.

If these considerations lead the players to prefer not using these conventional techniques, it would appear that the only choice left for the participants is to choose at random which strategy to pursue, which would lead to many instances of coordination failures and a consequent reduction in their respective payoffs. Nevertheless, as we now show, there is an alternative solution, which resorts to quantum mechanics to solve coordination problems without communication, trusted third parties or prearranged choices. It thus provides an additional option for addressing coordination problems, with a different set of strengths and limitations from those of the conventional approaches.

2. QUANTUM MECHANISM FOR COORDINATION

Quantum dynamics allows for the practical solution of a coordination problem via the generation of particles in entangled states. Quantum entanglement results in the appearance of specific quantum correlations between parts of a composite system, which can be exploited for quantum information processing.[16]

In the simplest case, where players face two choices, A, and B, they can use entangled particles with two physically observable states, such as their spin or polarization. Figure 1 schematically shows how the states are generated and distributed to the players. At a time of their choosing, each participant observes the state of their particle, resulting in either A or B as the outcome, and makes the corresponding choice. The key aspect that makes this technique different from random choices is that entanglement implies a definite correlation between the two measurements, i.e., both players get either outcome A or B, irrespective of the spatial separation between them, and without communication.

Thus, quantum information processing, which already offers the potential for improved computation, cryptography and economic mechanisms[3,6–8,12,14] can help solve coordination problems without resorting to the complex signaling procedures required to achieve common knowledge when limited to unreliable or asynchronous communication.

Mathematically, the quantum state is specified by a vector of complex numbers, called amplitudes, one for each possible configuration of the physical system. These amplitudes have physical meaning in that their squared magnitudes correspond to the probabilities of obtaining a given configuration when the state is observed. Any such vector can always be written as a linear combination of basis states. In this case, the basis state corresponding to configurations C of the system, is conventionally written as $|C\rangle$. In games for two players each having two choices (A or B), the configurations of the game are all possible joint choices. For a quantum implementation of a game, each player is given a physical state which can take on either of the two choices. Thus the whole game, i.e., choices for

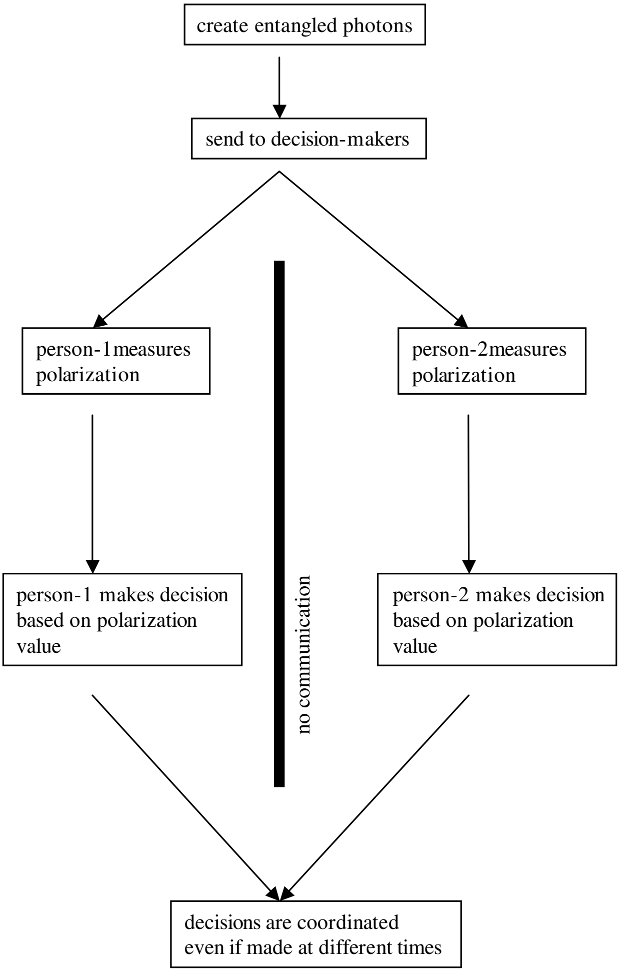

Fig. 1. Schematic description of the quantum coordination mechanism. A source of entangled photons sends one to each of the participants of a coordination game, who then observe it in order to make a decision.

all the players, is described by a quantum state vector. For the case of two players and two choices, we can illustrate the behavior of this mechanism with the vector:

$$|S\rangle = a|AA\rangle + a|BB\rangle + b|AB\rangle + b|BA\rangle \tag{1}$$

with the constants, a, b, subject to the normalization condition $2|a|^2 + 2|b|^2 = 1$ because the probabilities must sum to 1. Except for special choices of the constants (e.g., $a = b$), such a state is *entangled*, in the sense that it cannot be written as a product of individual players' states. This property of entanglement is the key feature that allows quantum coordination.

To make use of such a state, players can observe their part of the state by performing a physical measurement (e.g., the polarization of a photon). This is possible because the players have physical possession of the physical object representing their individual choices. For example, suppose the first player makes an observation of the state in Eq. (1). The outcome will be either A or B, with corresponding probabilities $|a|^2 + |b|^2 = 1/2$. This act of observation necessarily alters the overall state of the system, giving either:

$$a\sqrt{2}|AA\rangle + b\sqrt{2}|AB\rangle,$$
$$a\sqrt{2}|BB\rangle + b\sqrt{2}|BA\rangle,$$

when the first player observes A or B, respectively. When the other player makes an observation, the result will be the same as that of the first player with probability $2|a|^2$, and opposite with probability $2|b|^2$. We thus see that if the players desire to have the same choice, they should prepare Eq. (1) with $a = 1/\sqrt{2}$ and $b = 0$. Notice that while the outcomes are correlated, the individual choices are still completely random (i.e., each happens with probability 1/2), a property that cannot be achieved with a classical system in absence of communication or a priori agreement on specific choices or deterministic methods for generating them from common knowledge.

Notice that by using the quantum mechanism in a competitive setting the players can have undetectable randomness in their choices, no communication between them and still maintain complete correlation at every period of the game. Entanglement thus offers a way for the players to get correlated random bits they can use in addition to any public, broadcast information, without communication or prior agreement on the specific choices they will make. In this context, we should note that this mechanism does not allow the players to act completely independently: they must arrange to obtain the entangle pair and agree on how they will interpret their subsequent observations as choices in the game.

A cooperative setting where our quantum mechanism could be useful is one in which each player does not know others involved in the game, or they wish to remain mutually anonymous and avoid communication. If the interested players are known to be members of a larger group,[1] and entangled states are easily distributed among all members of the group, those players interested in coordinating their activities can use the entangled states to ensure all players make the same choice.

3. DISCUSSION

In this paper, we showed how quantum entanglement provides an alternate technique for solving coordination problems. Unlike prior extensions of classical games to allow quantum operations as additional choices,(3,6,8,14) this mechanism retains the same choices as in the original classical game and instead utilizes quantum entanglement as an extra resource to aid the participants in their choices. Furthermore, this quantum solution cannot be achieved via a classical simulation since we are assuming an absence of any communication among the participants. This is unlike the situation with other quantum games proposals.(20)

As to the implementation of these mechanisms, these quantum solutions of coordination problems are not just a theoretical construct, as they can be implemented over relatively large distances. In particular, parametric-down conversion techniques can produce twin photons which are perfectly quantum correlated in time, space and often in polarization.(11) These photons can then be physically separated by many kilometers so that each participant gets one of the entangled particles. If the lifetime of the entangled state is long, each participant can then receive an entangled photon and perform a polarization measurement later, thus not having to communicate with each other during the whole procedure. On the other hand, if the lifetime of the entangled state is shorter than the period of the game, photons can be regenerated periodically, thereby requiring a transmission channel from the source to the participants (but not between the participants). In this case the advantage lies not in avoiding the possibility of blocked communication by an adversary, but in avoiding the detection of a coordinated solution and the direct communication among the participants (which is relevant when the participants wish to remain anonymous to the adversary). This makes for a feasible quantum solution to coordination problems that can be implemented with current technology, in contrast with most schemes for arbitrary quantum computation.

The practical difficulties in implementing any of these schemes depend on the number of players who need joined entangled states and the extent to which they can prearrange to get the suitable states. In the simplest case, as discussed above, there are two players known to each other and can make prior arrangements to obtain entangled pairs. A more complicated case involves a network (a quantum internet(13)) that delivers entangled states to those who wish to participate while avoiding detection of who else they are entangled with. This implies the ability to create and distribute large numbers of pairs.

In principle, this quantum coordination mechanism applies to more complicated games with correlated equilibria among n players.[2] In these cases more complicated entangled states allow finding these equilibria without communication or prior agreements. It can also produce higher correlations than is possible for a classical games.[15] However, creating entangled states among many players in intrinsically more difficult. While it has been recently shown[18] that it is possible to create entangled states of few particles, it will be technically difficult to use this mechanism for coordination problems involving many participants who require all to share the same choices. On the other hand, even pairwise coordination could be useful for larger groups in cases requiring less stringent coordination, e.g., just having coordination among all pairs of players or a hierarchy of coordinated choices made by successively larger groups of the participants but with each level of the hierarchy only involving two subgroups.

An interesting issue is the extent to which the incentive structure changes in the face of practical issues such as noise and decoherence. For example it may be necessary to test or purify the entanglement in order to store it over long times, which may reduce the advantages of not having any communication. This question of practical implementation arises even in conventional classical games because the basic assumptions and payoff may not reflect the full complication of real life. Even more significantly, fully rational behaviors predicted by game theory do not always describe the choices made in practice, raising interesting empirical questions of how well people can learn to exploit the capabilities of entanglement as an decision aid in practical coordination games.

Finally, this quantum approach to coordination games is more general than it may first appear, as it can also be applied to a variety of economic situations that involve achieving some or partial coordination among members of a group. One example is several groups participating in an auction in which the value of an item to a person depends on what others in their group get. For example imagine bidding for construction tools that members of a group share and the auction is for each item separately. In this case, the valuation depends on the complementarity of the goods that the whole group gets, rather than who in the group gets each item. While more complicated than the pure coordination game we discussed, because this problem also involves a bidding strategy, it still involves the need for the group to coordinate without signaling to other groups.

As we have shown, the utilization of simple properties of quantum states gives an alternative solution to coordination problems, one that does not require communication, trusted third parties involved in the decision making or prior commitment. It can also achieve common knowledge without complex protocols involved when participants are limited to asynchronous communication. An important property of entangled states is that, without communication, they can only be used to produce correlated *random* choices with pre-arranged probabilities, as is appropriate for mixed-strategy scenarios. They cannot substitute for communication, e.g., if players would wish for their correlated choices to depend on private knowledge obtained by one player but without needing to communicate that information to the others.

An interesting question is identifying practical instances of coordination problems that would benefit from an alternative to the conventional approaches, and whether the properties of the quantum mechanism would be useful in these cases. This is a particularly relevant economic question since this quantum solution is achievable with today's technology. More generally, this discussion shows that the benefits of developments in quantum information technology are not limited to improving existing tasks (e.g., computation speed or cryptography), but could also provide the basis for novel economic mechanisms with a different mix of information privacy, communication requirements and trust guarantees than is possible with conventional methods.

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