Variations on Error Diffusion:
Retrospectives and Future Trends

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Outline

• Introduction

• Grayscale error diffusion
  – Analysis and modeling
  – Enhancements

• Color error diffusion halftoning
  – Vector quantization with separable filtering
  – Matrix valued error filter methods

• Conclusion
Human Visual System Modeling

- Contrast at particular spatial frequency for visibility
  - Bandpass: non-dim backgrounds
    [Manos & Sakrison, 1974; 1978]
  - Lowpass: high-luminance office settings with low-contrast images
    [Georgeson & G. Sullivan, 1975]
  - Exponential decay [Näsänen, 1984]
  - Modified lowpass version
    [e.g. J. Sullivan, Ray & Miller, 1990]
  - Angular dependence: cosine function [Sullivan, Miller & Pios, 1993]
Grayscale Error Diffusion Halftoning

- Nonlinear feedback system
- Shape quantization noise into high frequencies
- Design of error filter key to quality

\[
\text{difference threshold} = \text{compute error} + e(m)\]

\[
\text{Error Diffusion}
\]

\[
\text{Spectrum}
\]
Analysis of Error Diffusion I

- **Error diffusion as 2-D sigma-delta modulation**
  [Anastassiou, 1989] [Bernard, 1991]

- **Error image** [Knox, 1992]
  - Error image correlated with input image
  - Sharpening proportional to correlation

- **Serpentine scan places more quantization error along diagonal frequencies than raster** [Knox, 1993]

- **Threshold modulation** [Knox, 1993]
  - Add signal (e.g. white noise) to quantizer input
  - Equivalent to error diffusing an input image modified by threshold modulation signal
Example: Role of Error Image

- Sharpening proportional to correlation between error image and input image [Knox, 1992]

Floyd-Steinberg (1976)

Jarvis (1976)
Analysis of Error Diffusion II

• **Limit cycle behavior** [Fan & Eschbach, 1993]
  – For a limit cycle pattern, quantified likelihood of occurrence for given constant input as function of filter weights
  – Reduced likelihood of limit cycle patterns by changing filter weights

• **Stability of error diffusion** [Fan, 1993]
  – Sufficient conditions for bounded-input bounded-error stability: sum of absolute values of filter coefficients is one

• **Green noise error diffusion** [Levien, 1993] [Lau, Arce & Gallagher, 1998]
  – Promotes minority dot clustering

• **Linear gain model for quantizer** [Kite, Evans & Bovik, 2000]
  – Models sharpening and noise shaping effects
Linear Gain Model for Quantizer

- **Extend sigma-delta modulation analysis to 2-D**
  - Linear gain model for quantizer in 1-D [Ardalan and Paulos, 1988]
  - Linear gain model for grayscale image [Kite, Evans, Bovik, 1997]

- **Error diffusion is modeled as linear, shift-invariant**
  - Signal transfer function (STF): quantizer acts as scalar gain
  - Noise transfer function (NTF): quantizer acts as additive noise
**Linear Gain Model for Quantizer**

\[
x(m) \rightarrow u(m) \rightarrow K_s \rightarrow b(m)
\]

\[x(m) + e(m) \rightarrow h(m) \rightarrow K_s \rightarrow n(m) \rightarrow b(m)
\]

\[
STF = \frac{B_s(z)}{X(z)} = \frac{K_s}{1 + (K_s - 1)H(z)}
\]

\[
NTF = \frac{B_n(z)}{N(z)} = 1 - H(z)
\]

Put noise in high frequencies

\[H(z)\text{ must be lowpass}
\]

Also, let \(K_s = 2\) (Floyd-Steinberg)

Pass low frequencies
Enhance high frequencies

Highpass response
(independent of \(K_s\))
Linear Gain Model for Quantizer

- Best linear fit for $K_s$ between quantizer input $u(m)$ and halftone $b(m)$

$$K_s = \arg \min_{\alpha} \sum_{m} (\alpha u(m) - b(m))^2$$

$$K_s = \frac{1}{2} \sum_{m} |u(m)| = \frac{1}{2} \frac{E\{|u(m)|\}}{E\{u^2(m)\}}$$

- Does not vary much for Floyd-Steinberg
- Can use average value to estimate $K_s$ from only error filter

<table>
<thead>
<tr>
<th>Image</th>
<th>Floyd</th>
<th>Stucki</th>
<th>Jarvis</th>
</tr>
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<tr>
<td>barbara</td>
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<td>3.62</td>
<td>3.76</td>
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<tr>
<td>mandrill</td>
<td>2.03</td>
<td>3.38</td>
<td>3.45</td>
</tr>
<tr>
<td>Average</td>
<td>2.03</td>
<td>3.94</td>
<td>4.37</td>
</tr>
</tbody>
</table>
Visual Quality Measures [Kite, Evans & Bovik, 2000]

- **Sharpening**: proportional to $K_s$
  
  Value of $K_s$: Floyd Steinberg < Stucki < Jarvis

- **Impact of noise on human visual system**
  
  Signal-to-noise (SNR) measures appropriate when noise is additive and signal independent

  Create unsharpened halftone $y[m_1,m_2]$ with flat signal transfer function using threshold modulation

  Weight signal/noise by contrast sensitivity function $C[k_1,k_2]$

  $$\text{WSNR (dB)} = 10 \log_{10} \frac{\sum_{k_1,k_2} |X[k_1,k_2] C[k_1,k_2]|^2}{\sum_{k_1,k_2} |(X[k_1,k_2] - Y[k_1,k_2]) C[k_1,k_2]|^2}$$

  Floyd-Steinberg > Stucki > Jarvis at all viewing distances
Enhancements I: Error Filter Design

- **Longer error filters reduce directional artifacts**
  [Jarvis, Judice & Ninke, 1976] [Stucki, 1981] [Shiau & Fan, 1996]

- **Fixed error filter design: minimize mean-squared error weighted by a contrast sensitivity function**
  - Assume error image is white noise [Kolpatzik & Bouman, 1992]
  - Off-line training on images [Wong & Allebach, 1998]

- **Adaptive least squares error filter** [Wong, 1996]

- **Tone dependent filter weights for each gray level**
  [Eschbach, 1993] [Shu, 1995] [Ostromoukhov, 1998] [Li & Allebach, 2002]
Example: Tone Dependent Error Diffusion

- **Train error diffusion weights and threshold modulation**
  
  [Li & Allebach, 2002]

---

Enhancements
Enhancements II: Controlling Artifacts

• Sharpness control
  – Edge enhancement error diffusion [Eschbach & Knox, 1991]
  – Linear frequency distortion removal [Kite, Evans & Bovik 1991]
  – Adaptive linear frequency distortion removal
    [Damera-Venkata & Evans, 2001]

• Reducing worms in highlights & shadows
  [Eschbach, 1993] [Shu, 1993] [Levien, 1993] [Eschbach, 1996] [Marcu, 1998]

• Reducing mid-tone artifacts
  – Filter weight perturbation [Ulichney, 1988]
  – Threshold modulation with noise array [Knox, 1993]
  – Deterministic bit flipping quant. [Damera-Venkata & Evans, 2001]
  – Tone dependent modification [Li & Allebach, 2002]
Example: Sharpness Control in Error Diffusion

- **Adjust by threshold modulation** [Eschbach & Knox, 1991]
  - Scale image by gain $L$ and add it to quantizer input
  - Low complexity: one multiplication, one addition per pixel

\[ L = \frac{1}{K_s} - 1 = \frac{1 - K_s}{K_s} \quad \text{since } K_s \geq 1 \]

- **Flatten signal transfer function** [Kite, Evans & Bovik, 2000]
Original

Results

Enhancements

Floyd-Steinberg

Edge enhanced

Unsharpened
Enhancements III: Clustered Dot Error Diffusion

• **Feedback output to quantizer input** [Levien, 1993]

• **Dot to dot error diffusion** [Fan, 1993]
  – Apply clustered dot screen on block and diffuse error
  – Reduces contouring

• **Clustered minority pixel diffusion** [Li & Allebach, 2000]

• **Block error diffusion** [Damera-Venkata & Evans, 2001]

• **Clustered dot error diffusion using laser pulse width modulation** [He & Bouman, 2002]
  – Simultaneous optimization of dot density and dot size
  – Minimize distortion based on tone reproduction curve
Enhancements

Example #1: Green Noise Error Diffusion

- **Output fed back to quantizer input** [Levien, 1993]
  - Gain $G$ controls coarseness of dot clusters
  - Hysteresis filter $f$ affects dot cluster shape

$$x(m) + u(m) + e(m) \rightarrow b(m)$$
Example #2: Block Error Diffusion

- **Process a pixel-block using a multifilter**
  [Damera-Venkata & Evans, 2001]
  - FM nature controlled by scalar filter prototype
  - Diffusion matrix decides distribution of error in block
  - In-block diffusions constant for all blocks to preserve isotropy
Enhancements

**Results**

- Block error diffusion
- DBF quantizer

**Green-noise**

- Tone dependent
Color Monitor Display Example (Palettization)

- **YUV color space**
  - Luminance (Y) and chrominance (U,V) channels
  - Widely used in video compression standards
  - Contrast sensitivity functions available for Y, U, and V

- **Display YUV on lower-resolution RGB monitor:**
  use error diffusion on Y, U, V channels separably

\[ y(m) = x(m) + u(m) + h(m) \]

\[ b(m) = 12-bit \ RGB \ monitor \]

\[ 24-bit \ YUV \ video \]

\[ 12-bit \ RGB \ monitor \]
Vector Quantization but Separable Filtering

- **Minimum Brightness Variation Criterion (MBVC)**  
  [Shaked, Arad, Fitzhugh & Sobel, 1996]
  - Limit number of output colors to reduce luminance variation
  - Efficient tree-based quantization to render best color among allowable colors
  - Diffuse errors separably

\[
x(m) + \text{MBVC} \rightarrow u(m) \rightarrow \text{VQ} \rightarrow b(m)
\]

\[
h(m) + e(m) \rightarrow + \rightarrow \text{VQ} \rightarrow b(m)
\]
Color Error Diffusion

Results

Separable Floyd-Steinberg

Original

MBVC halftone
Non-Separable Color Halftoning for Display

• Input image has a vector of values at each pixel (e.g. vector of red, green, and blue components)

Error filter has matrix-valued coefficients

Algorithm for adapting matrix coefficients based on mean-squared error in RGB space

[Akarun, Yardimci & Cetin, 1997]

• Optimization problem

Given a human visual system model, find color error filter that minimizes average visible noise power subject to diffusion constraints [Damera-Venkata & Evans, 2001]

Linearize color vector error diffusion, and use linear vision model in which Euclidean distance has perceptual meaning

\[ t(m) = \sum_{k \in \Omega} \frac{\tilde{h}(k)}{\text{matrix}} \frac{e(m-k)}{\text{vector}} \]
Matrix Gain Model for the Quantizer

- Replace scalar gain w/ matrix [Damera-Venkata & Evans, 2001]

\[
\hat{K}_s = \arg \min_{\hat{A}} E \left( \left\| \mathbf{b}(m) - \hat{A} \mathbf{u}(m) \right\|^2 \right) = \tilde{C}_{bu} \tilde{C}_{uu}^{-1}
\]

\[
\hat{K}_n = \mathbf{I}
\]

- Noise uncorrelated with signal component of quantizer input
- Convolution becomes matrix–vector multiplication in frequency domain

**Grayscale results**

\[
B_n(z) = (\mathbf{I} - \tilde{H}(z)) N(z)
\]

\[
B_s(z) = \tilde{K} \left( \mathbf{I} + \tilde{H}(z)(\tilde{K} - \mathbf{I}) \right)^{-1} X(z)
\]

\[
\begin{align*}
(1 - H(z)) N(z) \\
\frac{K_s X(z)}{1 + (K_s - 1)H(z)}
\end{align*}
\]

Noise component of output

Signal component of output
**Linear Color Vision Model**

- **Undo gamma correction to map to sRGB**
- **Pattern-color separable model** [Poirson & Wandell, 1993]
  - Forms the basis for Spatial CIELab [Zhang & Wandell, 1996]
  - Pixel-based color transformation

![Diagram of Color Vision Model]

- Opponent representation
- B-W
- R-G
- B-Y
- Spatial filtering

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*Color Error Diffusion*
Color Error Diffusion

Example

Separable Floyd-Steinberg  Original  Optimum vector error filter
Evaluating Linear Vision Models
[Monga, Geisler & Evans, 2003]

- An objective measure is the improvement in noise shaping over separable Floyd-Steinberg

- Subjective testing based on *paired comparison task*
  - Observer chooses halftone that looks closer to original
  - Online at www.ece.utexas.edu/~vishal/cgi-bin/test.html
Subjective Testing

• **Binomial parameter estimation model**
  – Halftone generated by particular HVS model considered better if picked over another 60% or more of the time
  – Need 960 paired comparison of each model to determine results within tolerance of 0.03 with 95% confidence
  – Four models would correspond to 6 comparison pairs, total $6 \times 960 = 5760$ comparisons needed
  – Observation data collected from over 60 subjects each of whom judged 96 comparisons

• **In decreasing subjective (and objective) quality**
  Linearized CIELab $> >$ Opponent $> YUV \geq YIQ$
Grayscale & color halftoning methods
1. Classical and user-defined screens
2. Classical error diffusion methods
3. Edge enhancement error diffusion
4. Green noise error diffusion
5. Block error diffusion

Additional color halftoning methods
1. Minimum brightness variation
2. Quadruple error diffusion

Figures of merit for halftone evaluation
1. Peak signal-to-noise ratio (PSNR)
2. Weighted signal-to-noise ratio (WSNR)
3. Linear distortion measure (LDM)
4. Universal quality index (UQI)
**Selected Open Problems**

- **Analysis and modeling**
  - Find less restrictive sufficient conditions for stability of color vector error filters
  - Find link between spectral characteristics of the halftone pattern and linear gain model at a given graylevel
  - Model statistical properties of quantization noise

- **Enhancements**
  - Find vector error filters and threshold modulation for optimal tone-dependent vector color error diffusion
  - Incorporate printer models into optimal framework for vector color error filter design
Backup Slides
Need for Digital Image Halftoning

• Examples of reduced grayscale/color resolution
  – Laser and inkjet printers
  – Facsimile machines
  – Low-cost liquid crystal displays

• Halftoning is wordlength reduction for images
  – Grayscale: 8-bit to 1-bit (binary)
  – Color displays: 24-bit RGB to 8-bit RGB
  – Color printers: 24-bit RGB to CMY (each color binarized)

• Halftoning tries to reproduce full range of gray/color while preserving quality & spatial resolution
  – Screening methods are pixel-parallel, fast, and simple
  – Error diffusion gives better results on some media
Screening (Masking) Methods

- Periodic array of thresholds smaller than image
  - Spatial resampling leads to aliasing (gridding effect)
  - Clustered dot screening produces a coarse image that is more resistant to printer defects such as ink spread
  - Dispersed dot screening has higher spatial resolution
  - Blue noise masking uses large array of thresholds
Basic Grayscale Error Diffusion

Introduction
Compensation for Frequency Distortion

• **Flatten signal transfer function** [Kite, Evans, Bovik, 2000]
  \[
  L = \frac{1 - K_s}{K_s} \quad (L \in (-1, 0]) \text{ since } K_s \geq 1
  \]

• **Pre-filtering equivalent to threshold modulation**
  \[
  G(z) = 1 + L(1 - H(z)) \quad \text{FIR filter}
  \]
Block FM Halftoning Error Filter Design

- FM nature of algorithm controlled by scalar filter prototype
- Diffusion matrix decides distribution of error within a block
- In-block diffusions are constant for all blocks to preserve isotropy

\[
\Gamma = \gamma \otimes \tilde{D} \quad \tilde{D} \text{ diffusion matrix}
\]

\[
\tilde{D} = \frac{1}{N^2} [\mathbf{1}] \quad N \text{ is the block size}
\]
Linear Color Vision Model

• **Undo gamma correction on RGB image**

• **Color separation** [Damera-Venkata & Evans, 2001]
  – Measure power spectral distribution of RGB phosphor excitations
  – Measure absorption rates of long, medium, short (LMS) cones
  – Device dependent transformation \( C \) from RGB to LMS space
  – Transform LMS to opponent representation using \( O \)
  – Color separation may be expressed as \( T = OC \)

• **Spatial filtering included using matrix filter** \( \tilde{d}(m) \)

• **Linear color vision model**
  \[
  \tilde{v}(m) = \tilde{d}(m)T \quad \text{where} \quad \tilde{d}(m) \quad \text{is a diagonal matrix}
  \]
Designing the Error Filter

- Eliminate linear distortion filtering before error diffusion
- Optimize error filter $h(m)$ for noise shaping

$$\min E\left[ \|b_n(m)\|^2 \right] = E\left[ \| \tilde{v}(m) * (\mathbf{I} - \mathbf{h}(m)) * n(m) \|^2 \right]$$

Subject to diffusion constraints

$$\sum_m \mathbf{h}(m) \mathbf{1} = 1$$

where $\tilde{v}(m)$ linear model of human visual system
$*$ matrix-valued convolution
Generalized Optimum Solution

- Differentiate scalar objective function for visual noise shaping w/r to matrix-valued coefficients

\[
\frac{d\left\{ E\left[ \| b_n(m) \|^2 \right] \right\}}{dh(i)} = 0 \quad \forall i \in \mathcal{O} \quad \| x \| = Tr(xx')
\]

- Write norm as trace and differentiate trace using identities from linear algebra

\[
\frac{d\left\{ Tr(\tilde{A}\tilde{X}) \right\}}{d\tilde{X}} = \tilde{A}' \\
\frac{d\left\{ Tr(\tilde{X}'\tilde{A}\tilde{X}\tilde{B}) \right\}}{d\tilde{X}} = \tilde{A}\tilde{X}\tilde{B} + \tilde{A}'\tilde{X}\tilde{B}'
\]

\[
\frac{d\left\{ Tr(\tilde{A}\tilde{X}\tilde{B}) \right\}}{d\tilde{X}} = \tilde{A}'\tilde{B}'
\]

\[
Tr(\tilde{A}\tilde{B}) = Tr(\tilde{B}\tilde{A})
\]
Generalized Optimum Solution (cont.)

- Differentiating and using linearity of expectation operator give a generalization of the Yule-Walker equations

\[ \sum_k \tilde{v}'(k) \tilde{r}_{an} (-i - k) = \sum_p \sum_q \sum_s \tilde{v}'(s) \tilde{v}(q) \tilde{h}(p) \tilde{r}_{nn} (-i - s + p + q) \]

where

\[ a(m) = \tilde{v}(m) \ast n(m) \]

- Assuming white noise injection

\[ r_{nn}(k) = E[n(m) n'(m + k)] \approx \delta(k) \]

\[ r_{an}(k) = E[a(m) n'(m + k)] \approx \tilde{v}(-k) \]

- Solve using gradient descent with projection onto constraint set
Implementation of Vector Color Error Diffusion

\[
\tilde{H}(z) = \begin{pmatrix}
H_{rr}(z) & H_{rg}(z) & H_{rb}(z)
H_{gr}(z) & H_{gg}(z) & H_{gb}(z)
H_{br}(z) & H_{bg}(z) & H_{bb}(z)
\end{pmatrix}
\]

\[
\begin{pmatrix}
r \\
g \\
b
\end{pmatrix}
\]

\[
\begin{pmatrix}
H_{gr} \\
H_{gg} \\
H_{gb}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\times \\
\times
\end{pmatrix}
\]
Generalized Linear Color Vision Model

- **Separate image into channels/visual pathways**
  - Pixel based linear transformation of RGB into color space
  - Spatial filtering based on HVS characteristics & color space
  - Best color space/HVS model for vector error diffusion?
    [Monga, Geisler & Evans 2002]
Linear CIELab Space Transformation

[Flohr, Kolpatzik, R.Balasubramanian, Carrara, Bouman, Allebach, 1993]

- **Linearized CIELab using HVS Model by**

  \[ Y_y = 116 \frac{Y}{Y_n} - 116 \quad L = 116 f\left(\frac{Y}{Y_n}\right) - 116 \]

  \[ C_x = 200 [\frac{X}{X_n} - \frac{Y}{Y_n}] \quad a = 200 [ f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right) ] \]

  \[ C_z = 500 [\frac{Y}{Y_n} - \frac{Z}{Z_n}] \quad b = 500 [ f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right) ] \]

  where

  \[ f(x) = \begin{cases} 
  7.787x + 16/116 & 0 \leq x \leq 0.008856 \\
  (x)^{1/3} & 0.008856 \leq x \leq 1 
  \end{cases} \]

- **Linearize the CIELab Color Space about D65 white point**

  Decouples incremental changes in \( Y_y, C_x, C_z \) at white point on \( (L,a,b) \) values

  \[ \nabla_{(Y_y,C_x,C_z)} (L,a,b) = (1/3)I \]

  \( T \) is sRGB \( \rightarrow \) CIEXYZ \( \rightarrow \) Linearized CIELab
Spatial Filtering

• **Opponent** [Wandell, Zhang 1997]
  – Data in each plane filtered by 2-D separable spatial kernels
    \[ f = k \sum_i w_i E_i \]
    \[ E_i = k_i \exp\left[-\frac{(x^2 + y^2)}{\sigma_i^2}\right]. \]
  – Parameters \((w_i, \sigma_i)\) for the three color planes are

<table>
<thead>
<tr>
<th>Plane</th>
<th>Weights (w_i)</th>
<th>Spreads (\sigma_i)</th>
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<td></td>
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<td>Red-green</td>
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<td>Blue-yellow</td>
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<td></td>
<td>0.371</td>
<td>0.386</td>
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</tbody>
</table>
Spatial Filtering

- Spatial Filters for Linearized CIELab and YUV,YIQ based on:
  
  Luminance frequency Response [Nasanen and Sullivan – 1984]

  \[ W_{(Y_y)}(\tilde{p}) = K(L) \exp[-\alpha(L)\tilde{p}] \]

  \[ L - \text{average luminance of display}, \tilde{p} \text{ the radial spatial frequency and} \]

  \[ \alpha(L) = \frac{1}{c \ln(L) + d} \quad K(L) = aL^b \quad \tilde{p} = \frac{p}{s(\phi)} \]

  where \( p = (u^2 + v^2)^{1/2} \) and

  \[ s(\phi) = \frac{1 - w}{2} \cos(4\phi) + \frac{1 + w}{2} \]

  \( w - \text{symmetry parameter} = 0.7 \) and \( \phi = \arctan\left( \frac{v}{u} \right) \)

  \( s(\phi) \) effectively reduces contrast sensitivity at odd multiples of 45 degrees which is equivalent to dumping the luminance error across the diagonals where the eye is least sensitive.
Chrominance Frequency Response [Kolpatzik and Bouman – 1992]

\[ W_{(C_x, C_z)}(p) = A \exp[-\alpha p] \]

Using this chrominance response as opposed to same for both luminance and chrominance allows more low frequency chromatic error not perceived by the human viewer.

- The problem hence is of designing 2D-FIR filters which most closely match the desired Luminance and Chrominance frequency responses.
- In addition we need zero phase as well.

*The filters (5 x 5 and 15 x 15 were designed using the frequency sampling approach and were real and circularly symmetric).*

Filter coefficients at: [http://www.ece.utexas.edu/~vishal/halftoning.html](http://www.ece.utexas.edu/~vishal/halftoning.html)

- Matrix valued Vector Error Filters for each of the Color Spaces at [http://www.ece.utexas.edu/~vishal/mat_filter.html](http://www.ece.utexas.edu/~vishal/mat_filter.html)
Color Spaces

• Desired characteristics
  – Independent of display device
  – Score well in perceptual uniformity [Poynton color FAQ http://comuphase.cmetric.com]
  – Approximately pattern color separable [Wandell et al., 1993]

• Candidate linear color spaces
  – Opponent color space [Poirson and Wandell, 1993]
  – YIQ: NTSC video  Eye more sensitive to luminance; reduce chrominance bandwidth
  – YUV: PAL video
  – Linearized CIELab [Flohr, Bouman, Kolpatzik, Balasubramanian, Carrara, Allebach, 1993]
Monitor Calibration

• **How to calibrate monitor?**
  sRGB standard default RGB space by HP and Microsoft
  Transformation based on an sRGB monitor (which is linear)

• **Include sRGB monitor transformation**
  \[ T: \text{sRGB} \rightarrow \text{CIEXYZ} \rightarrow \text{Opponent Representation} \]
  [Wandell & Zhang, 1996]
  Transformations sRGB \rightarrow YUV, YIQ from S-CIELab Code
  at http://white.stanford.edu/~brian/scielab/scielab1-1-1/

• **Including sRGB monitor into model enables Web-based subjective testing**
  http://www.ece.utexas.edu/~vishal/cgi-bin/test.html
Spatial Filtering

• **Opponent** [Wandell, Zhang 1997]
  Data in each plane filtered by 2-D separable spatial kernels
  \[ f = k \sum_i w_i E_i \]
  \[ E_i = k_i \exp\left[-\left(\frac{x^2 + y^2}{\sigma_i^2}\right)\right]. \]

• **Linearized CIELab, YUV, and YIQ**
  Luminance frequency response [Näsänen and Sullivan, 1984]
  \[ W_{(Y_y)}(\rho) = K(L) e^{-\alpha(L)\rho} \]
  \(L\) average luminance of display
  \(\rho\) radial spatial frequency
  Chrominace frequency response [Kolpatzik and Bouman, 1992]
  \[ W_{(C_x,C_z)}(\rho) = A e^{-\alpha \rho} \]
  Chrominance response allows more low frequency chromatic
error not to be perceived vs. luminance response