Applying Evolutionary Game Theory to Auction Mechanism Design

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auctions, e-commerce, game theory, evolution, genetic algorithms, multi-agent systems

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ABSTRACT

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Keywords

Auctions, Economics, e-commerce

1. INTRODUCTION

Auctions are an important class of mechanisms for resolving multi-agent allocation problems of various types. There exists a substantial body of work (see, e.g. [14] for a review) regarding the theory underlying auctions, most of which focuses on the problem of how to design them so as to achieve some desired outcome for the auctioneer. In situations where the auctioneer plays the role of seller, this outcome is often revenue maximization, and many results of a qualitative nature are known regarding the suitability of different mechanisms under different assumptions on the economic scenario under consideration.

In parallel with this work, researchers have begun investigating how to design autonomous agents capable of participating in auctions ([18], [1], [4], [6], [7], etc.). Often such study is motivated by the possibility that suitable autonomous agents will be superior to humans in making (possibly quite complex) economic decisions, and indeed Das et al. report human experiments to substantiate this possibility [11]. When the agents acting in markets are non-human, the space of potential market designs increases markedly, since mechanisms that might seem "non-sensical" or difficult to interpret for humans can be considered.

Recently, a few papers have begun to address the confluence of these ideas, taking inspiration from the Auction Theory work on mechanism design, extending it into new design spaces that might have been infeasible before, and adding a degree of automation to the design process: for example, in [8] and [9], Cliff describes an application of Genetic Algorithms [13] to the choice of a continuous parameter Q_s governing the probability that a seller will be chosen to shout in a given round of a stylized continuous double auction.

This paper continues the of such work, examining a space of auction mechanisms that includes the standard first- and second-price auctions, using GAs applied to a multi-agent system to evolve good players for each mechanism under consideration. We find that under several classes of non-pathological conditions (e.g. bidders are risk-averse, and are unaware of how many players they will face in a given auction), there exist exotic sealed bid mechanisms which are expected to return significantly higher revenue to the auctioneer than either the first- or second-price sealed bid mechanisms. See Section 4 for more details.

The paper is laid out as follows: in the next two sections we discuss the methods used, introducing relevant Game Theory concepts as needed. In Section 4 we describe the results of our experiments. In Section 5 we describe in more detail the relationship between these results and others in the literature, and we conclude in Section 6.

2. AUCTION THEORY

2.1 Terms and Notation

In this paper we study sealed-bid auctions, in which a good is put up for sale, and each potential buyer submits a bid to the auctioneer; the auctioneer chooses a winner, and allocates payments to each agent. In most variants of this type of auction, the good is awarded to the buyer who submits the highest bid, and only the winner pays. In a **first-price** auction, the winner's payment is equal to her bid; in a **second-price** auction, the winner's payment is equal to the second highest bid.

In order to analyse how bidders might be expected to behave in such auctions, we need to specify how they are motivated, i.e. what is the good worth to each agent. We use a model in which agents are only interested in their own awards and payments, and there is some intrinsic monetary "value" v associated to the good. All outcomes can therefore be represented by a single number, the monetary gain the agent makes. This is $v - x$ for a win with payment x, and $-x$ for a non-win with the same payment. The risk preferences of agents are differentiated by use of a von Neumann – Morgenstern utility function u, so that an agent strictly prefers a selection of possible outcomes x_i with corresponding probabilities p_i, over a second selection of possible outcomes y_j with corresponding probabilities q_j, if and only if

$$\sum_i p_i u(x_i) > \sum_j q_j u(x_j). \quad (1)$$

In this representation, assuming twice-differentiability of u, an agent for which $u''(x) = 0$ is known as **risk-neutral**; if $u''(x) < 0$, the agent is **risk-averse**, and if $u''(x) > 0$, the agent is known as **risk-seeking** (see Figure 1).

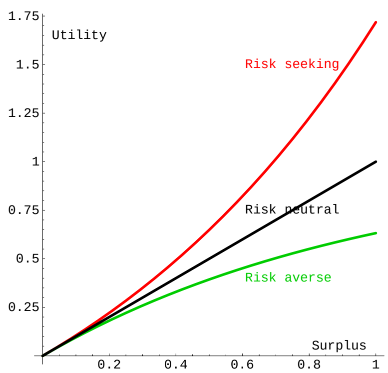

Figure 1: Typical utility functions for a risk- seeking, neutral and averse agent.

The value of a good to an agent can be independent of the value of the good to other agents, or it can be derived from information about how other agents value the good. In the former case we say that the agent has a **private** value for the good, and in the latter case that there is some **common** value component. We treat both these cases by postulating that each bidder receives a "signal", and that the value of the good to the agent is some specified function of *all* the agents' signals. Since a bidder only necessarily knows her own signal, her decision problem may in general involve guess work about the worth of the good.

2.2 Revenue for the Auctioneer

In this paper we study mechanisms from the point of view of the amount of money they are expected to make the seller. Perhaps the most important result in this area is the **Revenue Equivalence Theorem**, which states that if (concerning the environment)

- there is a fixed number of bidders, known to everyone,
- all agents are risk-neutral,
- all bidders' signals are picked from a common, known distribution,

and if (concerning the mechanism),

- in equilibrium, the good always goes to the bidder with the highest signal,
- any bidder whose signal is the lowest possible expects to make nothing,

then the expected revenue to the seller is the same, independent of the mechanism.

This rather surprising result means that, subject to these hypotheses, it *doesn't matter* what type of auction a seller runs, he should expect to make the same amount of money whatever the mechanism. But of course there *are* many different auction mechanisms in use, of extremely variable type, because at least one of the hypotheses on which the Revenue Equivalence Theorem rests is often violated. It is known, for example, that most people are not risk-neutral[1], and in the case when the bidders are risk-averse, it makes more sense for a seller to run a first-price auction.

A method commonly used to establish an ordering on auctions for different types of buyer, is to treat the problem as a non-cooperative game, and solve for the game's Nash equilibrium. The difficulty with doing this is that the equations used to define such an equilibrium might well be intractible. In this paper we pursue an alternative method for determining an ordering: we *simulate* a population of buyers, and play the game many times with random selections from this population. The resulting averaged returns for the seller are estimates of the true expected return. As such, the results they give are not as satisfactory as those derived from Game Theory.

3. METHODS

The basic methodology pursued was to instantiate a group of agents according to various environmental and agent-preference parameters, and let them compete in a specified auction according to specified strategies, logging the utility extracted by each agent as a result. Since many of the environmental parameters require some degree of randomization in the agent instantiation (e.g. randomized private value for the good), this procedure was repeated a large number of times, so as to generate an estimate of the expected utility to an agent of using a given strategy in a given context.

3.1 Context Parameterization: Environment, Preferences and Mechanism

We chose to investigate a space of mechanisms very similar to the first- and second-price sealed bid auctions specified earlier.

Definition 1. *Let $w = (w_1, \ldots, w_n)$ be a vector of n real numbers. A w-price auction is a sealed bid auction in which the highest bidder wins the good, and pays*

$$\frac{\sum_{j=1}^{N} w_j bid_j}{\sum_{j=1}^{N} w_j} \quad (2)$$

[1]Most people tend to act in a risk-averse manner in their daily lives.

where N is the minimum of n and the number of bidders, and $bid_1, bid_2, \ldots$ are the bids, ordered highest to lowest.

In this paper we examine a one-dimensional sub-space of w-price auctions, namely those of type $w = (1-w_2, w_2)$. In this parameterization, $w_2 = 0$ is a standard first price auction, $w_2 = 1$ a standard second-price auction, and all other values of w_2 correspond to non-standard auction types that have not previously been studied.

The space of agent preferences and environmental variables which we explored was motivated by examining exceptions to the Revenue Equivalence Theorem; we allowed variable group size, variable risk preference, and correlated (non-independent) bidders' signals. In addition, we allowed the degree of commonality in values to be altered.

1. The number of agents in each trial was either an arbitrary fixed number, or was chosen with uniform probability from a set of consecutive integers bigger than 2. In most experiments the fixed number was chosen to be 6, and the range $\{2, 3, 4, 5, 6\}$.

2. The signals $(t_1, \ldots, t_n)$ of a group $(a_1, \ldots, a_n)$ of bidders were chosen to be a weighted sum of a shared random signal and a sequence of independent random signals, with each such signal coming from a uniform distribution on $[0, 1]$[2]. Thus independent variables S, $X_1, \ldots, X_n$ were generated, and the signal t_i for agent a_i was chosen to be $c\,S + (1-c)X_i$, where $c \in [0, 1]$ parameterizes the degree of correlation between agents' signals.

3. The calculation of the utility extracted from winning the good depends on two properties of the agents involved: their risk preferences, and the degree of commonality in value. For risk, we chose to use Constant Absolute Risk utility functions:

$$u_\alpha(x) = \begin{cases} \frac{1}{\alpha}(e^{\alpha x} - 1) & \text{if } \alpha \neq 0, \\ x & \text{if } \alpha = 0, \end{cases} \tag{4}$$

   α is zero for risk-neutral agents, negative for risk-averse agents and positive for risk-seeking agents. Figure 1 plots these functions for $\alpha = -1, 0, 1$.

   To model common value, we assumed that the monetary value to agent a_i of winning the good was given by $d \cdot \left(\sum_j v_j\right)/n + (1-d)v_i$, where d is a parameter controlling the degree of common value, with $d = 0$ representing purely private values, and $d = 1$ purely common values. Thus the utility reward to agent a_i of winning the good at bid b, conditional on signals t_j for all players, was

$$u_\alpha\left(d\Big(\sum_j t_j\Big)/n + (1-d)v_i - b\right). \tag{5}$$

[2] We also performed experiments with the family of distributions

$$B_{m,k}(x) = m \cdot x^{m-k}(1-x)^k \binom{m-1}{k-1} \tag{3}$$

(for $k = 0, \ldots, m$). The results generated with these distributions were not qualitatively different from those generated for the uniform distribution.

3.2 Strategy Optimization

The above variables w_2, c, d, α specify the context in which the agents have to act, but not how they should act in that context. The most challenging piece of analysis in Mechanism Design is always figuring out how a bidder is likely to behave. The standard Game Theory approach is to enumerate all strategies that an agent might pursue, and determine a strategy *from which deviation is not rational*, i.e. which is expected-utility maximizing given that the other agents' behaviour is fixed.

As mentioned before, this process is often impossible, either because of the intractability of the strategy space, or because the equations which need to be solved to determine a deviation-proof strategy are too complex.

In this paper we take an empirical approach to finding good strategies, whereby each agent in a population of bidders is equipped with a bidding function which can be modified through evolution to adapt to the necessities of the game. As the agents play the game, successful strategies are bred preferentially, and thus the entire population improves. There is constant pressure to improve, because if an agent's deviation from the norm gives it a slightly higher expected utility, then it will be slightly more likely to breed than average, and so its genes will be preferentially reproduced into the next generation.

The main drawback to this approach is that it can neither be guaranteed that the population will evolve a good strategy within a reasonable period of time, nor that the solution on which the population eventually coverges is a global rather than local optimum. Thus we gain formal simplicity at the cost of computation. We run the entire process of evolution many times independently, and reduce the effect of mutation as time goes by, so as to encourage convergence.

The link between genomes and bidding function was as follows: A gene consisted of a sequence $(g_1, g_2, \ldots, g_k)$ of real-valued "control points" assigned to evenly spaced input signals $(0, 1/k, \ldots, 1)$. The bid output for an input signal t was generated by interpolating between the control points:

$$bid(t) = \begin{cases} g_1 & \text{if } t < 0 \\ g_l + (k.t - l)(g_{l+1} - g_l) & \text{if } t \in [l/k, (l+1)/k) \\ g_{k+1} & \text{if } t \geq 1 \end{cases} \tag{6}$$

This representation was chosen over others (e.g. power series representation, GP etc.) because it combines useful features of the domain, while placing very few restrictions on the space of all such functions. Specifically,

1. Stability: These functions are stable under small random mutations: changing the data c_i does not make a

huge difference to the output values generated by the bid function.

2. Locality: A change in a value g_l has no effect on the function for signals above $(l+1)/n$ or below $(l-1)/n$, so each g value, or sequence of g values, represents a partial solution for a certain input range.

In addition, the functions generated are guaranteed to be continuous.

These data g_l were then mutated and recombined according to a standard Genetic Algorithm, where the fitness of a given genome was determined relative to other genomes by participation in a sequence of randomized games. Specifically, the evaluation of a population of genomes was according to the following algorithm:

```
For each of a large number of iterations {
    while (not all agents have played in this round) {
        select some as-yet-unplayed agents to play a game
        generate random signals for the agents
        get bids for each agent, according to their genome
        select a winner and determine payments
        accumulate the corresponding utility rewards
    }
}
```

An agent's fitness was equal to its accumulated utility from all the games. This process is modular with respect to the contextual parameters specified in Section 3.1.

The Genetic algorithm was simply

```
generate a population of N random genomes
for each generation {
    assess the fitness of every individual by
        playing a large number of games as above
    rank the genomes by this fitness measure
    repeat N times {
        select two genomes from the old population,
            favouring highly ranked genomes
        select a random point in the genome, and
            combine the first half of one with the
            second half of the other
        with pre-selected probability, mutate each
            of the genes by an amount picked from a
            pre-selected distribution[3]
        place this new genome into the new population
    }
}
```

Thus we did not necessarily preserve the fittest individual from each generation. Notice that the fitness function is stochastic, so genomes can gain unfair advantage from being lucky (in the selection of their signals, for example). Notice also that the fitness is measured relative to other agents. This means that the most successful agent strategy is not necessarily that which gives greatest expected return, since it may (for example) be incentive compatible in such an environment to deliberately disadvantage oneself if in doing so ones opponents are even more disadvantaged. An example of this is that agents with very low signals are (in most environments) incented to bid higher than their valuation: they are very unlikely to win the good (and hence have negative surplus), whereas they are much more likely to decrease the winner's surplus, and hence increase their own relative fitness.

Thus this process finds good players at the repeated competitive game, not at the one-shot game. It is hoped (and we shall demonstrate, in some cases) that this effect is very small, so that conclusions about the one shot game can still be made; it is worth noting that in real auctions with real players (humans or corporations), exogenous effects such as these are commonplace.

[3]The mutation probability was constant (at 0.8), and the maximal size of the mutation μ was reduced as time went on, being given, in generation G, by $\mu = \mu_0(G_0/(G_0+G))^\lambda$, for fixed $\mu_0 = 0.05$, $G_0 = 20$, and $\lambda = 1.5$

4. RESULTS

Shown below (Figure 2) is a graph showing the genes of the best individual in a population of 360, plotted against generation number, for an example evolutionary run for risk-neutral agents with independent private values, competing in a second price auction. In this context, the optimal bidding strategy is to bid one's signal, so given that $k = 5$ (i.e. the genome consists of 5 control values), the expected-utility-maximizing genome is $(0, 0.25, 0.5, 0.75, 1.0)$. As can be seen, the best individual is initially far from perfect, and varies greatly over the first few generations, since mutation is (relatively) high, and the population very diverse. As time wears on, however, the population discovers a bidding strategy that is close to optimal: the final population's best individual's bid is always within 5% of its optimal value.

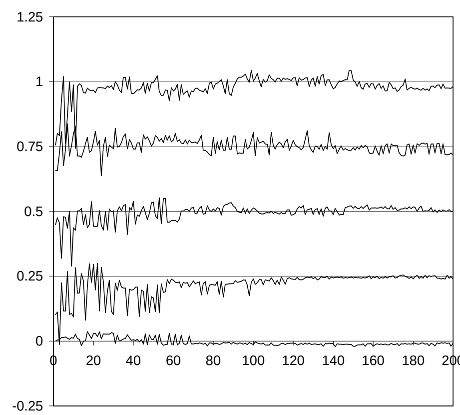

Figure 2: Genome for best individual in an example evolutionary run, plotted as a function of generation number.The population is initialized to random rational strategies (i.e. agents in the initial population cannot lose money, initially).

We first verified the method by calculating revenue landscapes for situations where the ordering of first and second price auctions is qualitatively known. When players are risk-neutral, signals independent, and the number of players fixed, the Revenue Equivalence Theorem says that all our mechanisms will generate the same expected revenue. Figure 3 plots expected seller revenue against w_2 (in black). These results were obtained by evolving a population of 360 agents for 200 generations, 200 times, and taking the average auctioneer revenue across these 200 trials. The revenue was always calculated on the basis of all agents using the best individual strategy from generation 200. The two lines in grey, above and below the plotted curve of average revenue, are plus and minus one standard deviation relative to the average, and give an indication of the magnitude of experimental uncertainty.

As can be seen, there is no experimentally significant difference in revenue between any of the mechanisms in the risk-neutral independent private values case.

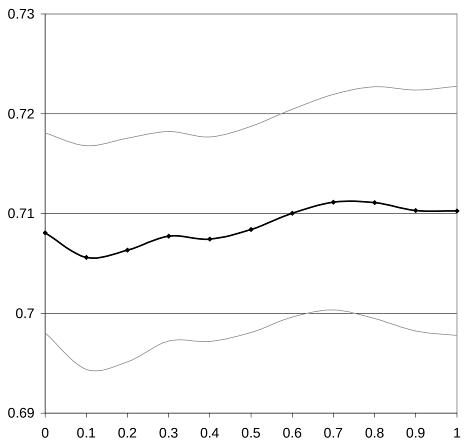

Figure 3: Sample revenues for risk-neutral agents operating in groups of fixed size 6, versus coefficient w_2 of second-highest bid in payment. Left-hand side is first price auction, right-hand side is second-price auction.

As mentioned in Section 2.2, when we modify the above by having risk-averse buyers, the first price auction becomes preferred. Figure 4 shows this effect.

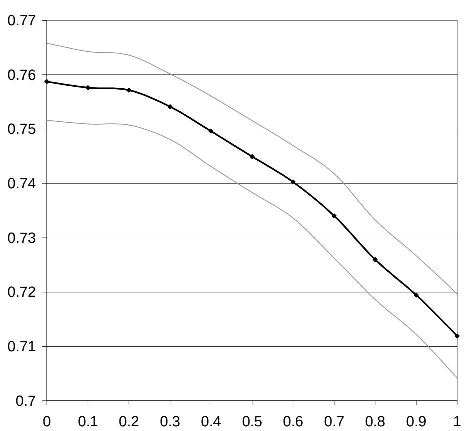

Figure 4: Average revenues for risk-averse agents operating in groups of fixed size 6.

As Milgrom et al. demonstrate, when values are correlated[4], we expect that the second-price auction will give greater revenue [15]. Figure 5 demonstrates this effect occurring.

Much more interesting than confirming known results, is

[4]In fact [15] discuss not correlation but *affiliation* between bidders' signals. It can be shown that the joint signal distributions we have chosen to use in this paper satisfy this stronger affiliation condition.

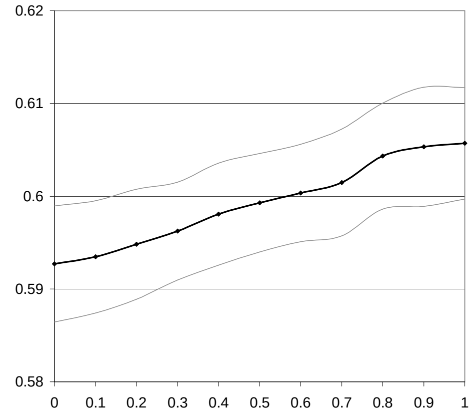

Figure 5: Average revenues for risk-neutral agents in fixed-size groups with values that are 50% correlated.

investigating regions of the environment space where there are no clear cut results. For example, if buyers have partial common values, and are risk averse, then either first or second price could be optimal for the seller, depending on the magnitude of the two effects.

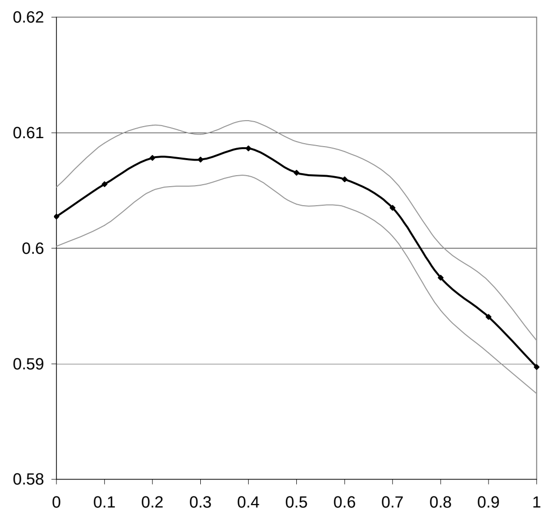

Figure 6: Average revenues for risk-averse agents ($\alpha = -10$) in fixed-size groups with values that are 50% common.

Figure 6 shows the situation when bidders are risk-averse, with parameter -10, and have a common value coefficient of 0.5. In this case, the first-price auction is clearly superior to the second-price auction. More surprisingly, a $(0.3, 0.7)$-price auction is superior to both first- and second-price auctions.

Figure 7 shows the same situation when the common value coefficient is 0.9, and risk aversion is -15. In this case, second-price is superior to first-price, and once again a w-price auction is superior to both. These auction forms, in which the winner pays a weighted average of his own and the second player's bid are not studied in the literature, but in this common scenario, can be revenue maximizing, de-

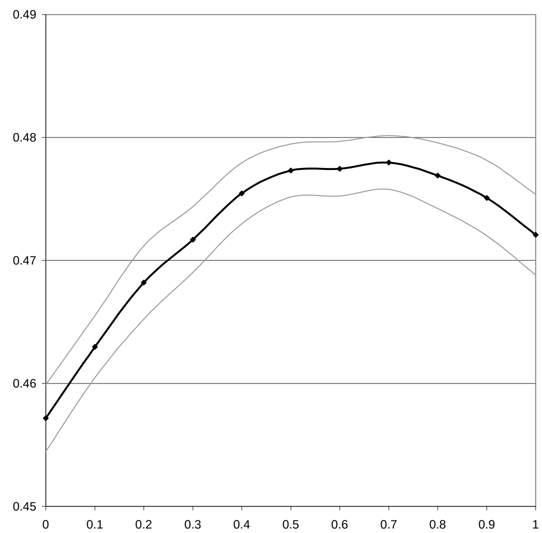

Figure 7: Average revenues for risk-averse agents ($\alpha = -15$) in fixed-size groups with values that are 90% common.

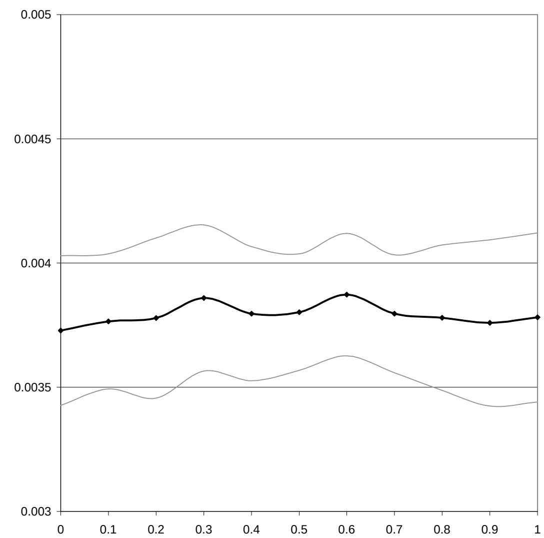

Figure 9: Graph of expected utility earned by each agent as a function of w_1 for the equilibrium strategy in the scenario described in Figure 7

pending on the nature of the agents playing the game. The optimality of non-standard auctions in this risk-averse, partial common-value setting persists if the number of agents is variable, as is seen in Figure 8.

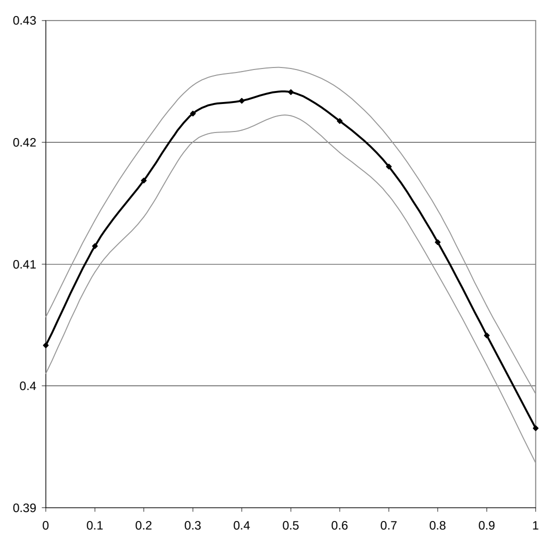

Figure 8: Average revenues for risk-averse agents ($\alpha = -15$) with values that are 90% common, when group size is random in the range $[2, 6]$.

In both of these scenarios, the graph of utility versus w_1 is flat (see Figure 9): the agents themselves are indifferent as to which auction they participate in. Thus selecting a revenue-maximizing value of w_1 need not antagonize bidders, however, as Bergman et al. show in [2], we should expect the real dynamics of auction choice on the part of bidders to be affected by more than just expected revenue: the variance of payments is crucial also. Clearly more work is needed to understand population dynamics in this new environment.

5. RELATED WORK

Auction Theory is a mature field, with a substantial literature. We shall not attempt an exhaustive review here; interested readers are referred to [14] for an overview.

The use of agents to investigate economic phenomena via simulation is a much newer field, known, broadly, as Agent-based Computational Economics [22], a sub-field of which is concerned with designing agents that can operate in on-line auctions or negotiations. See, for example [1], [6] with respect to 1-1 negotiation; [7], [19], [12], [11], [21] with respect to continuous double auctions; [17], [18], [5] with respect to sequences of English Auctions and [4], [3] with respect to sequences of sealed bid auctions.

When it comes to investigating novel auction types automatically, or semi-automatically, the citations are much thinner on the ground. A general discussion of automated mechanism design appears in [10], which deals with issues of computational complexity, but does not address any practical implementation details. The work of Cliff [8] is the first to provide a complete system for automated mechanism design. Cliff addresses the case of a continuous open-cry auction, using a Genetic Algorithm to adjust both the parameters of the bidding agents he uses, and the mechanism parameter, which in this case is the probability Q_s that in any given round, a seller will be chosen at random to make an offer. Besides being based on the continuous double auction, Cliff's work differs from ours in two significant ways. Firstly, the space of agent strategies explored is necessarily very restrictive[5], whereas the strategy space our GA explores contains close approximations to all continuous bidding strategies. Secondly, although Cliff's choice of mechanism space

[5]This problem difficult to address in the continuous open-cry auction, because such auctions have an intractably complicated strategy space: an agent will typically have many opportunities to act, for each of which the information space is the set of all previous actions by all agents.

was inspired by the experimental design used by Smith in [20], the Continuous Double Auction as it is used in such real-world institutions as the New York stock exchange is quite different – using order queues and bid improvement rules, for example; our cases $w_2 = 0, 1$ are faithful interpretations of first- and second-price sealed bid auctions, which are used in the world on a daily basis.

The work of Phelps et al. [16] provides another approach to modifications of the continuous double auction, in which the modification is to the clearing rule, via use of Genetic Programming.

6. CONCLUSIONS

In this paper we have described an application of simulated evolutionary game theory to a mechanism design problem. We have demonstrated that this technique can be used to explore a space of auction mechanisms, and by doing so in a specific setting that involves faithful versions of real-world mechanisms, have established the superiority of non-standard auction types in a variety of common environments.

The advantages of such a method for exploring auction design issues are clear: the agents discover good bidding strategies by evolution, without the need for complicated, possibly intractable, and certainly fragile mathematical analysis. In more complicated applications, the evolution process can implicitly take factors into consideration that might not have occurred to analysts. Additionally, the mechanism is tested for revenue generation against a small neighborhood of strategies, not just the Nash-equilibrium strategy. As a result, its sensitivity to agents' choice of strategy can be determined.

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