Deterministic, Analytically Complete Measurement of Polarization-Dependent Transmission Through Optical Devices

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Polarization dependence of the loss or gain of an optical device has been difficult to measure in a consistent and reproducible manner because it has been necessary to search for the extrema of transmission over a two-dimensional polarization space. It is shown for the first time that the global variation of the transmission through any linear, time-invariant optical device, over all states of polarization, can be found in a strictly deterministic, analytically complete manner by measuring the polarization responses to only three input polarizations. A series of fast, automated measurements of two test devices yielded standard deviations of 0.017 dB and 0.033 dB, and agreement with laborious manual measurements.
1 Introduction

Accurate, thorough characterization of optical devices is becoming increasingly important as optical systems become more complex and incorporate a wide variety of devices in ever larger numbers. One of the fundamental specifications of any device with an optical input and an optical output is polarization sensitivity of its transmission: the variation of optical power transmitted through the device as the input state of polarization (SOP) is varied. For example, the splitting ratio and excess loss of a fiber-optic directional coupler, the insertion loss of an optical isolator, and the gain of an optical amplifier all can exhibit variation as the input SOP is changed. In order to use any such device effectively in most practical optical systems, the polarization sensitivity of its transmission and/or reflection characteristics must be known. Measurement of polarization sensitivity is especially important when a system is made up of many concatenated optical devices, when selection of components with minimal polarization sensitivity is often desirable.

At the present state of the art, polarization sensitivity is directly measured by monitoring the output power of an optical device under test (DUT) with a polarization-independent detector or optical power meter while the input SOP is deliberately varied over all possible values, as shown in Fig. 1. Many devices have been devised to transform a fixed input SOP into any desired output SOP; we will refer to such a device as a polarization transformer (PT). Two independently-rotatable quarter-wave plates constitute a PT suitable for an optical beam propagating through open space, and two or more single-mode fiber loops of variable orientation can serve as a PT in fiber systems [1]. Both of these PTs are manually driven and do not lend themselves to automation, but alternatives exist which can be electronically controlled. For example, fiber-based PTs based on strain-induced birefringence brought about through piezoelectric or electromagnetic elements have been demonstrated, as have PTs based on electrooptic crystals or waveguides [2].

Regardless of the implementation of the PT, any measurement system similar to that shown in Fig. 1 suffers two fundamental disadvantages. One is the difficulty of arbitrary SOP generation, as the control inputs to a PT do not relate to the output SOP in a simple way, especially as the input SOP or source wavelength varies. Moreover, the output intensity of the PT is usually a weak function of the control inputs, and this variability in intensity translates directly into errors in measurement of the polarization dependence of transmission. These uncertainties in SOP and intensity can be ameliorated by monitoring the output of the PT with a polarimeter. A second, more serious disadvantage is the necessity of a search algorithm. At constant power, the state of a purely polarized source (i.e. with unit degree of polarization) has two degrees of freedom, so the SOP at the output of the PT must be varied over a two-dimensional space while searching for the global minimum and maximum transmission. Again, a polarimeter to monitor the output of the PT may be a practical necessity. The search algorithm must not mistakenly identify local extrema of transmission as the desired global values. Other sources of error include polarization sensitivity of the power meter, unstable source amplitude or unstable device transmission, and, unless a separate polarizer is used as shown in Fig. 1, partial polarization of the source.
We here demonstrate for the first time, both theoretically and experimentally, that the global maximum and minimum transmission through a device over all SOPs can be determined without resorting to a search over polarization space. The polarization responses to only three input SOPs completely determine the polarization sensitivity of a linear, time-invariant optical device.

2 Theory

R. C. Jones gave an explicit algorithm for experimentally determining the forward transmission Jones matrix $T$ of an unknown linear, time-invariant optical device [3]. The restriction of linearity precludes optical devices that generate new optical frequencies different from those of the input signal. The restriction of time invariance applies only to the polarization transformation caused by the device, and does not include the absolute optical phase delay. Therefore, this technique can be used to characterize pigtailed optical devices even when the phase delay through the fiber pigtails is drifting during the measurement.

We begin by generating a stimulus optical field of linear polarization parallel to the $x$ axis, then measure the resulting response Jones vector $h$. Similarly, stimulus fields of linear polarization parallel to the $y$ axis, and parallel to the bisector of the angle between the positive $x$ and $y$ axes result in response Jones vectors $v$ and $q$, respectively. Three complex ratios independent of the intensities of the three stimulus fields can now be formed from the $x$ and $y$ components of $h$, $v$, and $q$: $k_1 = h_x / h_y$, $k_2 = v_x / v_y$, and $k_3 = q_x / q_y$. A fourth ratio $k_4 = (k_3 - k_2) / (k_1 - k_3)$ is then found. To within a complex constant $\beta$, the transmission Jones matrix $T$ is then given [3] by

$$T = \beta \begin{bmatrix} k_1 k_4 & k_2 \\ k_4 & 1 \end{bmatrix}.$$  

We can define a standard inner product $(x, y) = y^t x$ to associate a scalar with any pair of complex Jones vectors $x$ and $y$, where $y^t$ is used to denote the complex-conjugate transpose of $y$. The intensity of an optical field represented by the Jones vector $x$ is then proportional to the inner product $(x, x)$. The field of values of a square matrix $A$ is defined as the set of complex numbers $(Ax, x)$ where $x$ ranges over all vectors that are normalized so that $(x, x) = x^t x = 1$. It can be shown [4] that the field of values of a Hermitian matrix is an interval of the real line, and that the eigenvalues of a Hermitian matrix are real. Furthermore, the minimum and maximum of the field of values of a two-by-two Hermitian matrix $H$ are given by its eigenvalues $\lambda_i(H)$, where $i = 1, 2$. The singular values $s_i$ of $A$ are given by $s_i(A) = \lambda_i((A^t A)^{1/2})$. Singular values are nonnegative real numbers. It can be shown [4] that $s_i^2(A) = \lambda_i(A^t A)$, and that the singular values of a
square matrix are invariant under unitary transformation, i.e. for any Jones matrix $A$ and any two unitary Jones matrices $U$ and $V$, $s_i(A) = s_i(VAU)$.

To measure polarization sensitivity, one must find the minimum and maximum intensity transmission coefficients $T_{\text{min}}$ and $T_{\text{max}}$ through a device over all possible SOPs. If the input field to the device is given by the Jones vector $x$, the output field can be written $Ax$ and the problem is reduced to finding the extrema of the inner product $(Ax, Ax)$ over all inputs $x$ of a constant intensity $(x, x)$. Because $(Ax, Ax) = (Hx, x)$ where $H = A^{*t}A$ is Hermitian, finding the minimum and maximum intensity transmission coefficients is equivalent to finding the extrema of the field of values of $H$. Thus, $T_{\text{min}}$ and $T_{\text{max}}$ are given by the eigenvalues of $H$, or equivalently, by the squares of the singular values of $A$.

Suppose the only access to the device under test is through fiber pigtails (Fig. 2). While direct measurement of the device matrix $A$ is then impossible, the matrix $B = VAU$, where $U$ and $V$ are unitary Jones matrices representing the pigtails, is easily determined to within a complex scalar constant $\beta$ using Eq. (1) and the method previously described. Because its loss is independent of polarization, each pigtail can be represented by a unitary Jones matrix times a scalar [5]. Having measured $B' = \beta B$, we can compute the singular values $s_i(B') = |\beta| s_i(B)$. Finally, using the fact that singular values are invariant under unitary transformation, we obtain an expression for the polarization dependence of transmission in terms of the singular values of the measured Jones matrix $B'$:

$$\frac{T_{\text{min}}}{T_{\text{max}}} = \frac{s_1^2(A)}{s_2^2(A)} = \frac{s_1^2(B)}{s_2^2(B)} = \frac{s_1^2(B')}{s_2^2(B')}$$

3 Experimental results and discussion

Experimental demonstration of the singular value technique requires only a polarimeter and a polarization synthesizer able to generate a series of known SOPs. A simple polarization synthesizer, sufficient for this method, consisted of a circularly polarized collimated beam followed by three linear polarizers oriented at 0, 45 and 90 degrees, which were sequentially inserted into the beam. Since the Jones matrix given by equation (1) is independent of the intensities of the three stimulus fields, the polarization synthesizer can generate stimuli of unequal intensities with no deleterious effects, the only limitation being that enough optical power reach the polarimeter to allow an accurate measurement of the response SOPs. A real-time (>1000 samples/sec) polarimeter was used, and the complete singular value measurement time was limited to somewhat less than two seconds owing to the limited speed of the mechanical polarization synthesizer. The singular value technique is thus orders of magnitude faster than traditional polarization-searching techniques.

Experimental results comparing the manual method to the singular value method for measurement of the polarization sensitivity of two optical isolators with single-mode fiber pigtails are shown in Fig. 3. The shaded areas indicate the manually measured polarization
sensitivity of the two test devices using the apparatus of Fig. 1, including a measurement uncertainty of $\pm 0.03$ dB caused by residual polarization dependence of the power meter and slight drifts in the intensity of the polarized source. The singular value measurement was performed ten times for each device, with both the input and output polarization transformers shown in Fig. 2 randomly reset for each of the ten measurements to present a different unitary polarization transformation before and after the device each time a Jones matrix was measured. The tight grouping of the plotted measurement points experimentally verifies the invariance of the singular value ratio over a variety of unitary input and output transformations, demonstrating that the polarization transformations caused by the pigtails do not impair measurement accuracy.

Manual measurement (Fig. 1) of isolator A using a Fabry-Perot laser diode source yielded a polarization dependence of $1.07 \pm 0.03$ dB. Singular value measurements ranged from $1.04$ dB to $1.10$ dB, yielding a mean $\pm$ standard deviation of $1.07 \pm 0.017$ dB. Measurements of isolator B were performed using an edge-emitting LED source whose spectrum was truncated using a monochromator with a 5-nm bandwidth. Manual measurement of polarization dependence yielded $0.12 \pm 0.03$ dB, and singular value measurements ranged from $0.08$ dB to $0.18$ dB, yielding a mean $\pm$ standard deviation of $0.131 \pm 0.033$ dB. Excellent agreement between the two methods is apparent, confirming the accuracy of this fast, deterministic technique. Measurement of a Jones matrix and calculation of its singular values took less than two seconds.

4 Summary
A new technique has been demonstrated for measuring the variation in transmission of linear, time-invariant optical devices, over all SOPs, by measuring the polarization responses to three stimulus SOPs. For the first time, this technique provides a deterministic, analytically complete means to characterize the polarization sensitivity of devices such as isolators or directional couplers. By eliminating the search over polarization space it allows a completely specified test suitable for comparisons and standards. The measurements were performed in less than two seconds, and resulted in the same values of polarization dependence obtained by tedious manual measurement.

5 References
Fig. 1. Apparatus used in the traditional polarization-search technique for measurement of polarization-dependent transmission through a device under test. A search algorithm must be employed to find the global minimum and maximum transmission, without mistakenly identifying local extrema.
Fig. 2. Apparatus used in the singular value technique for automated measurement of polarization-dependent transmission. Measurement of three SOPs yields the ratio of maximum to minimum transmission, over all input SOPs, in a deterministic closed form. L: lens; P: linear polarizer; PT: polarization transformer
Fig. 3. Measurements of the polarization dependence of transmission of two pigtailed optical isolators. The shaded areas indicate the manually measured values including ±0.03 dB uncertainty. Ten singular value measurements of each isolator are plotted, each measured with different input and output birefringences (different settings of PT1 and PT2 in Fig. 2.)