The Poincaré Sphere

The Poincaré sphere is a graphical tool in real, three-dimensional space that allows convenient description of polarized signals and of polarization transformations caused by propagation through devices. Any state of polarization can be uniquely represented by a point on or within a unit sphere centered on a rectangular (x,y,z) coordinate system. The coordinates of the point are the three normalized Stokes parameters describing the state of polarization. Partially polarized light can be considered a combination of purely polarized light of intensity I_P and unpolarized light of intensity I_U.

The degree of polarization $I_P/(I_P+I_U)$ corresponding to a point is the distance of that point from the coordinate origin, and can vary from zero at the origin (unpolarized light) to unity at the sphere surface (completely polarized light). Points close together on the sphere represent polarizations that are similar, in the sense that the interferometric contrast between two polarizations is related to the distance between the corresponding two points on the sphere.

Orthogonal polarizations with zero interferometric contrast are located diametrically opposite on the sphere. As shown in Fig. 1, linear polarizations are located on the equator. Circular states are located at the poles, with intermediate elliptical states continuously distributed between the equator and the poles. Right-hand and left-hand elliptical states occupy the northern and southern hemispheres, respectively.

Because a state of polarization is represented by a point, a continuous evolution of polarization can be represented as a continuous path on the Poincaré sphere. For example, the evolution of polarization for light traveling through a waveplate or birefringent crystal is represented by a circular arc about an axis drawn through the two points representing the eigenmodes of the medium. (Eigenmodes are polarizations that propagate unchanged through the medium.) A path can also record the polarization history of a signal, for example in response to changing strain applied to a birefringent fiber.

The real, three-dimensional space of the Poincaré sphere surface is closely linked to the complex, two-dimensional space of Jones vectors (see page 28). Most physical ideas can be expressed in either context, the mathematical links between the two spaces having previously been established for dealing with angular momentum. The graphical Poincaré description allows for a more intuitive approach to polarization mathematics.

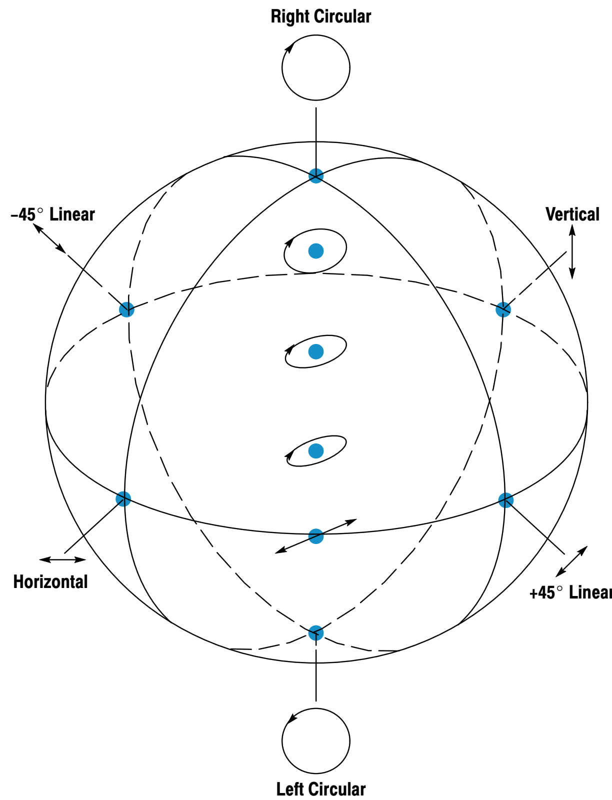

Fig 1. Points displayed on the surface of the Poincaré sphere represent the polarized portion of a lightwave. Linear polarizations are located on the equator. Circular states are located at the poles, with intermediate elliptical states continuously distributed between the equator and the poles. Right-hand and left-hand elliptical states occupy the northern and southern hemispheres, respectively. Example states are shown ascending the front of the sphere.

$$\langle\Delta\tau\rangle_{total} = \sqrt{\langle\Delta\tau\rangle_1^2 + \Delta\tau\rangle_2^2 + \dots}\,.$$

$ps/\sqrt{km}$

$\alpha\sqrt{8/\pi}$